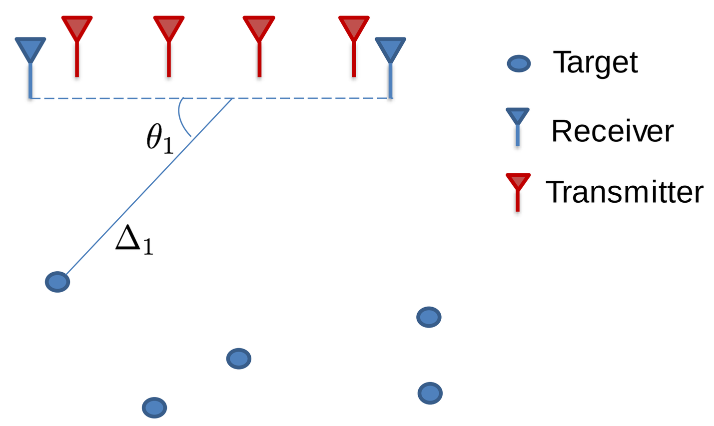
Waveform design for Compressive MIMO Radar

Nithin Sugavanam and Emre Ertin
Department of Electrical and Computer Engineering
The Ohio State University

Problem setup



- The MIMO radar with N_R and N_T collocated receivers and transmitters is analyzed
- Single pulse system is used to detect targets in the range-angle domain.
- The scene under investigation is assumed to consist of a small set of targets, which can be utilized to reduce the number of samples at the receiver.
- We present a waveform design for transmit elements when coupled with a standard stretch processing receiver can guarantee detection of K targets in a search space of S range-angle bins using $M = \mathcal{O}\left(\frac{K}{N_R}\log S\right)$ noisy measurements per receiver.

* The proposed design is better suited for practical implementation than the previously proposed designs based on random waveform.

Background

- The generic Radar signal model where each transmit element uses a waveform $s_i(t)$ and with K targets in the scene, the received signal at a particular receiver "l" is given as

$$y_l(t) = \sum_{k=1}^{K}\sum_{i=1}^{N_T} \alpha_R(\theta_k, l)\alpha_T(\theta_k, i) s_i(t-\Delta_k) x_k + w_l(t)$$

- The goal is to estimate $(\theta_k, \Delta_k, x_k)$ using noisy measurements This model can be discretized to get $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$.
- The target model considered is a K-sparse vector with support chosen uniformly at random with a uniformly distributed complex phase.
- Candes and Plan [2] state that support of K-sparse targets can be successfully recovered with high probability using LASSO if

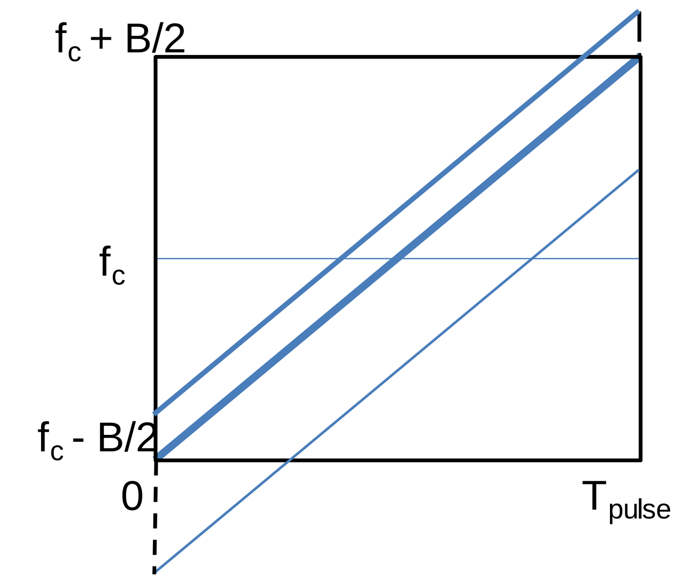
$$\mu(A) = \mathcal{O}\left(\frac{1}{logN}\right) \qquad K = \mathcal{O}\left(\frac{N}{\|A\|_{op}^2 \log N}\right)$$

$$\min_i |x_i| \geq 8\sigma\sqrt{2\log(NN_RN_T)}$$

- The mutual coherence of **A** is defined as $\mu(\mathbf{A}) = \max_{i,j} \frac{\langle A_i, A_j\rangle}{\|A_i\|\|A_j\|}$
- The operator norm of **A** is defined as it's largest singular value.

Measurement model

- The waveform employed by each of the transmitter is given as



$$s_i(t) = \frac{1}{\sqrt{N_c N_T}} \sum_{j=1}^{N_c} \exp\left(j2\pi((f_c + f_{i,j})t + \frac{B}{2\tau}t^2)\right)$$

- Stretch processing output due to K targets at receiver "l" in the scene is

$$y_l(t) = \frac{1}{\sqrt{N_R N_T N_c}} \sum_{k=1}^{K}\sum_{i=1}^{N_T}\sum_{j=1}^{N_c} \alpha_R(\theta_k, l)\alpha(\theta_k, i)\exp(j2\pi f_{i,j}\Delta_k)\exp\left(j2\pi(f_{i,j} - \frac{B}{\tau}\Delta_k)t\right) x_k + w_l(t)$$

- Parameter space of angle of arrival and range are divided into $N_R N_T$ and N grids based on the array's angular resolution and the Radar range resolution, respectively. The offset frequency is also discretized.
- M << N compressed samples in time are obtained at each of the receiver by employing a slow rate Analog to digital converter that is used to exploit the sparse structure in scene to recover the angle of arrival, range as well as the amplitude of the target.
- Sensing scheme can be compactly written as $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$, where $\mathbf{A} \in \mathbb{C}^{N_R M \times N_R N_T N}$ can be expressed as a random series of structured matrices.

Characterizing the sensing matrix

- The mutual coherence $\mu(\mathbf{A}) = \mathcal{O}\left(\sqrt{\frac{\log(N_R N_T N)}{M}}\right)$ with high probability.
- The operator norm $\|\mathbf{A}\|_{op} = \mathcal{O}\left(\sqrt{\frac{N_T N}{M}\log(N_R N_T N + N_R M)}\right)$ with high probability.

Support recovery guarantee

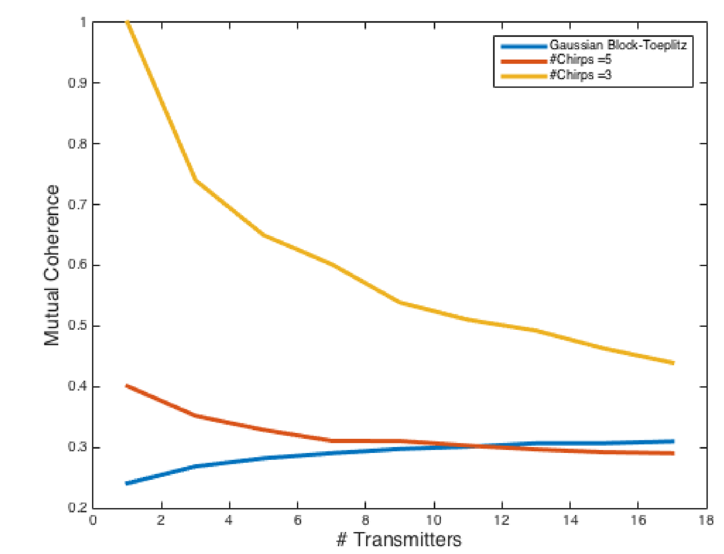
- For the proposed sensing scheme, the support or the target locations is recovered with high probability if, size of support

$$K \leq \frac{C N_R M}{\log(N_R N_T N)\log(N_R N_T N + N_R M)}$$

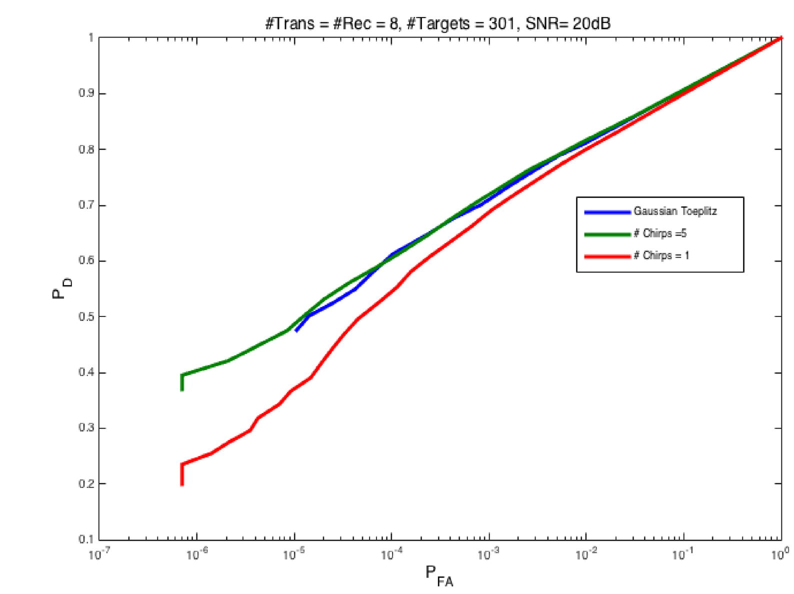
$$M \geq \log(N_R N_T N)^3 \qquad N_c N_T \geq \nu N, \nu \ll 1$$

$$\min_i |x_i| \geq 8\sigma\sqrt{2\log(NN_RN_T)}$$

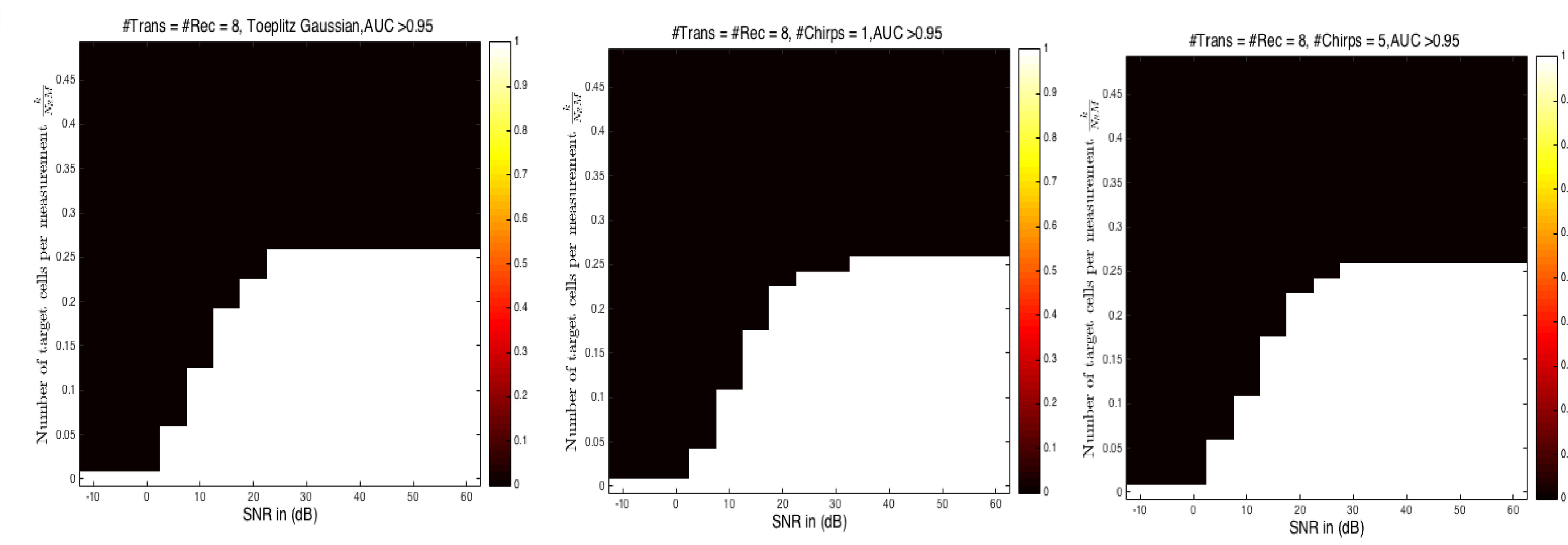
Mutual Coherence



Receiver operating characteristics



Phase transition for Area Under Receiver Operator Characteristics curve for support recovery



Future Work

- Design efficient algorithms for estimation of angle of arrival and range using the structured compressive measurements.
- Extend the results to recover sparse scene without the grid assumption

References

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[2] E. Candes and Y. Plan, "Near-ideal model selection by 1 minimization," *The Annals of Statistics*, vol. 37, no. 5A, pp. 2145–2177, Oct. 2009.
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[4] N. Sugavanam and E. Ertin, "Recovery guarantees for multifre-quency chirp waveforms in compressed radar sensing," *CoRR*, vol. abs/1508.07969, 2015.