Information Surrogates to Quantify the Value of Information for State Estimation

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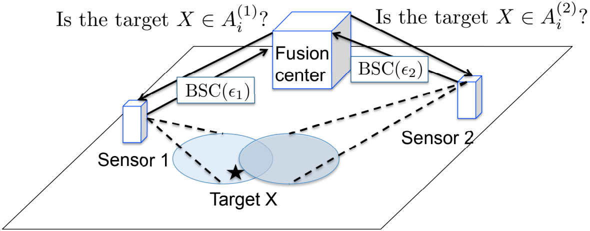
Value of Adaptivity and Information Surrogate

Problem Formulation: Noisy 20 questions game for target estimation in the setting of stochastic search to minimize $\mathbb{E}[|X - \hat{X}(Y_1^n)|^2]$



Information Surrogate: Minimizing conditional entropy $h(X|Y_1^n)$

- $\min_{\{A_1,\ldots,A_n\}} h(X|Y_1^n) \Leftrightarrow \max_{\{A_1,\ldots,A_n\}} I(X;Y_1^n)$
- Necessary condition: $\mathbb{E}[|X - \hat{X}(Y_1^n)|^2] \geq \frac{1}{2\pi e} e^{2 \cdot h(X|Y_1^n)}$

Questions:

1. Under the entropy proxy, what is the value of adaptivity?
2. Is $h(X|Y_1^n)$ the right information surrogate for mean squared error?

Value of Adaptivity Under Entropy Proxy

Value of Adaptivity: Both optimum adaptive and non-adaptive policies can achieve the maximum entropy reduction.

$$h(X|Y_1^n) \geq h(X) - nC \text{ where } C = 1 - H_B(\epsilon)$$

Mean Squared Error (MSE) Bound: For the optimum adaptive policy, $MSE \to 0$ as $n \to \infty$, but for the non-adaptive policy, it does not.

What Does the Conditional Entropy Fail to Capture?

Entropy is invariant to permutation $\Rightarrow$ It cannot capture *concentration of measure*

Question: Can we find a class of distributions such that variance is a monotonic function of entropy?

Entropy Proxy For Unimodal Distributions

Theorem (MSE and Conditional Entropy)

For unimodal distributions with exponentially decreasing tails, $p(X) = e^{-cx^{|\theta|}}/Z$ *where* $\theta > 1$,

$$\frac{e^{2\cdot h(X)}}{2\pi e} \leq \mathsf{Var}(X) \leq \frac{\alpha}{(\mathcal{I}(X))^{\beta}} \leq \frac{\alpha \cdot e^{2\beta\cdot h(X))}}{2\pi e}$$

for some $\alpha, \beta > 0$ *and Fisher information* $\mathcal{I}(X) = \mathbb{E}[\big(\frac{\partial}{\partial x}\log p(x)\big)^2]$.

$\Rightarrow$ For such unimodal distributions, any strategy that minimizes conditional entropy can achieve an exponentially decreasing MSE