Optimal Quantization of Likelihood for Low Complexity Sequential Testing

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Research Problem

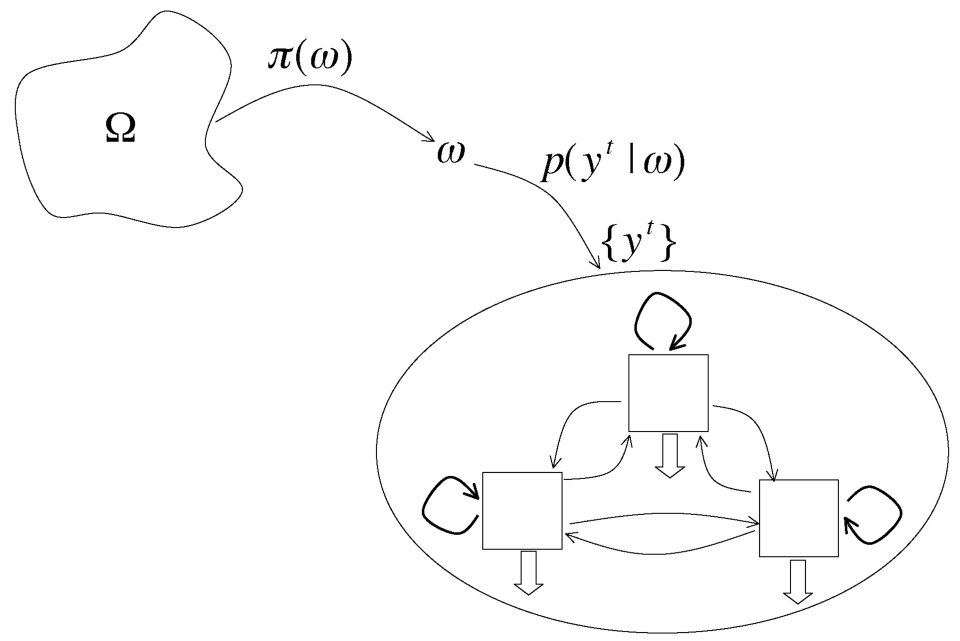

Figure 1: Sequential Testing with Quantized Beliefs

Spatially distributed set of micro sensor nodes can provide surveillance and monitoring of large scale structures. To detect and localize faults sensors have to integrate their information across time. Successful adoption of large scale distributed sensor systems is only possible if such networks can provide long lifetime. As a result temporal fusion of the sensor data has to be performed using low complexity/low power algorithms. Sequential decision procedures rely on computing, aggregating and communicating likelihood information at high precision which might be unsuitable for low power sensor nodes with limited computation and communication capability. Here, we consider design of quantized likelihood algorithms in the form of finite state machines suitable for implementation in low complexity devices.

Optimal Sequential Probability Ratio Test without Memory

In no memory case, we need to specify only the stopping rule and the final decision rule. The expected total cost for the test is:

$$C = \pi P_M + (1-\pi)P_F + c[(1-\pi)N(H_0) + \pi N(H_1)] \tag{1}$$

The optimal quantization rule in this case is given as follows:

$$\tilde{\pi} = \frac{\pi N(H_1)}{\pi N(H_1) + (1-\pi)N(H_0)} \quad . \tag{2}$$

And the boundaries conditions are calculated through the intersections of:

$$V(\pi(y^t)) = (1-\pi(y^t))(cN(H_0) + P_F) + \pi(y^t)(cN(H_1) + P_M) \tag{3}$$

and

$$U(\pi(y^t)) = \min\{\pi(y^t), 1-\pi(y^t)\} \quad . \tag{4}$$

We proved if the centroid condition and the intersections are calculated alternatively, they will converge to the optimal quantizer.

Optimal Sequential Probability Ratio Test with L Bits of Memory

For a sequential test with L bits of memory, the form of the test is similar to the test without memory, except for that the rejection region now needs to be further divided into 2^L subspaces. Therefore, optimality conditions should be calculated for the 2^L centroids and 2^L+1 boundaries. Starting from state k we have the expected cost as:

$$C^k = \pi P_M^k + (1-\pi)P_F^k + c[(1-\pi)N_k(H_0) + \pi N_k(H_1)] \tag{5}$$

We derived in this case that the centroid conditions are:

$$\tilde{\pi}^j = \frac{\pi N_k^j(H_1)}{\pi N_k^j(H_1) + (1-\pi)N_k^j(H_0)} \tag{6}$$

And the 2^L+1 boundaries conditions can be calculated as the turning points of the lower envelop of the following equations:

$$V^m(\pi^l(y^t)) = \pi^l(y^t)(cN_m(H_1) + P_M^m) + (1-\pi^l(y^t))(cN_m(H_0) + P_F^m) \tag{7}$$

and

$$U(\pi^l(y^t)) = \min\{\pi^l(y^t), 1-\pi^l(y^t)\} \tag{8}$$

We alternatively calculate the intersections and the centroids then repeat to get the optimal quantizer.

Results

In our simulation we compare our optimally designed agent to non-optimally designed agent and the ideal infinite memory agent in terms of expected test cost.

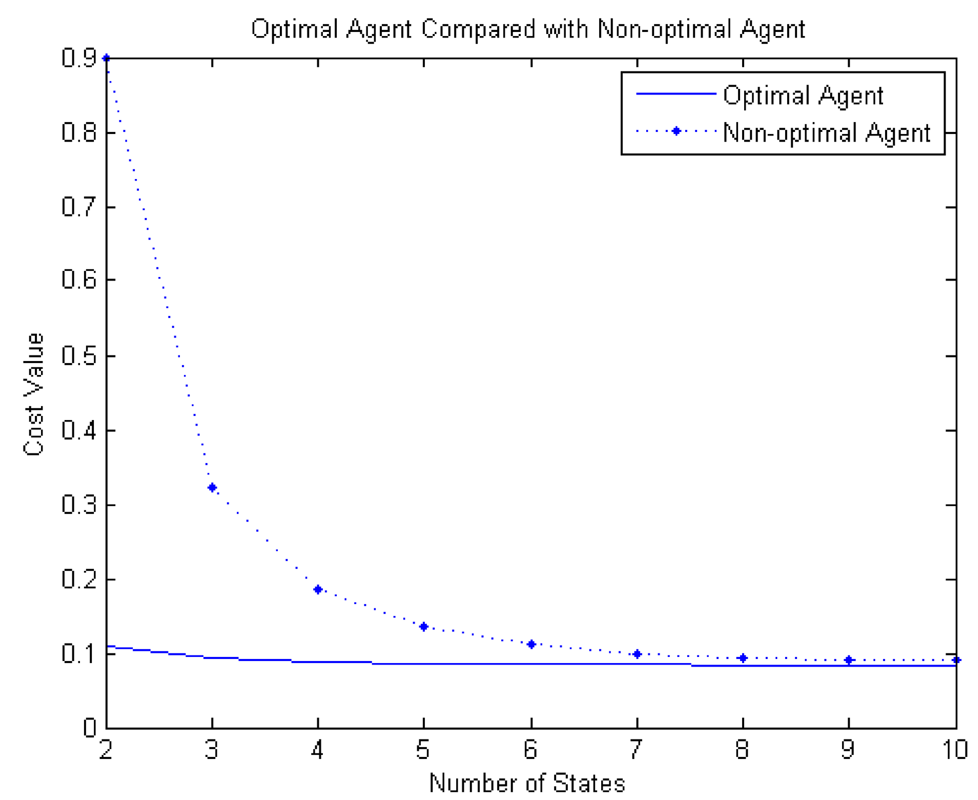

Figure 2: Optimal Agent Compared with Non-optimal Agent

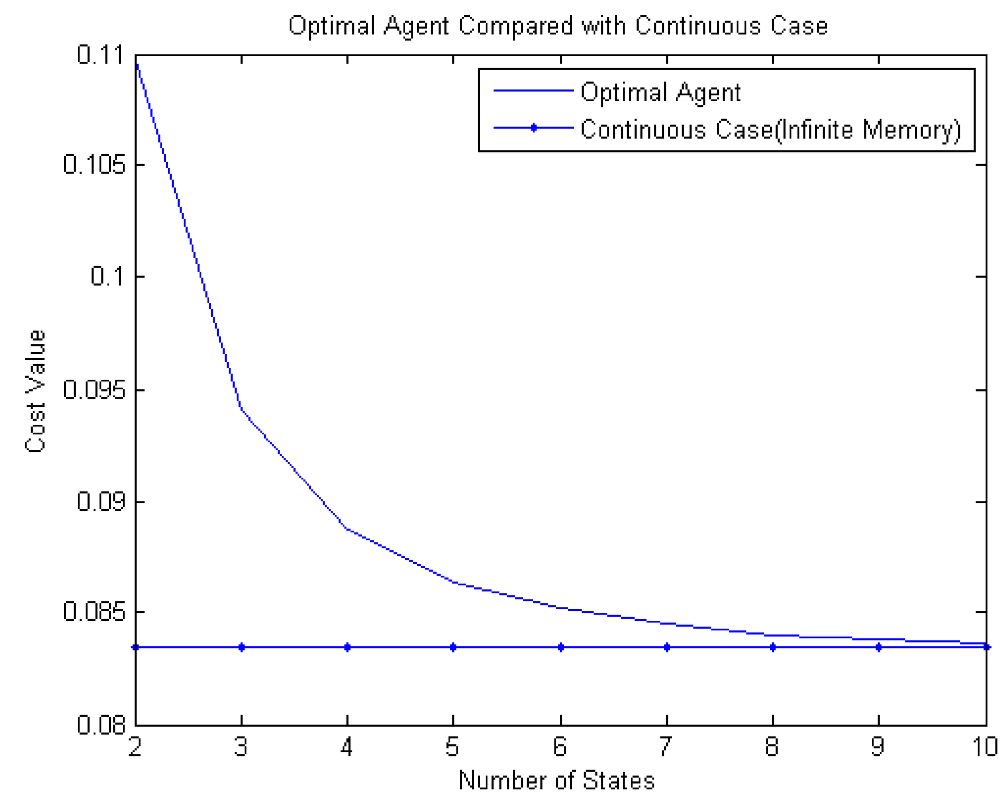

Figure 3: Optimal Agent Compared with Continuous Case

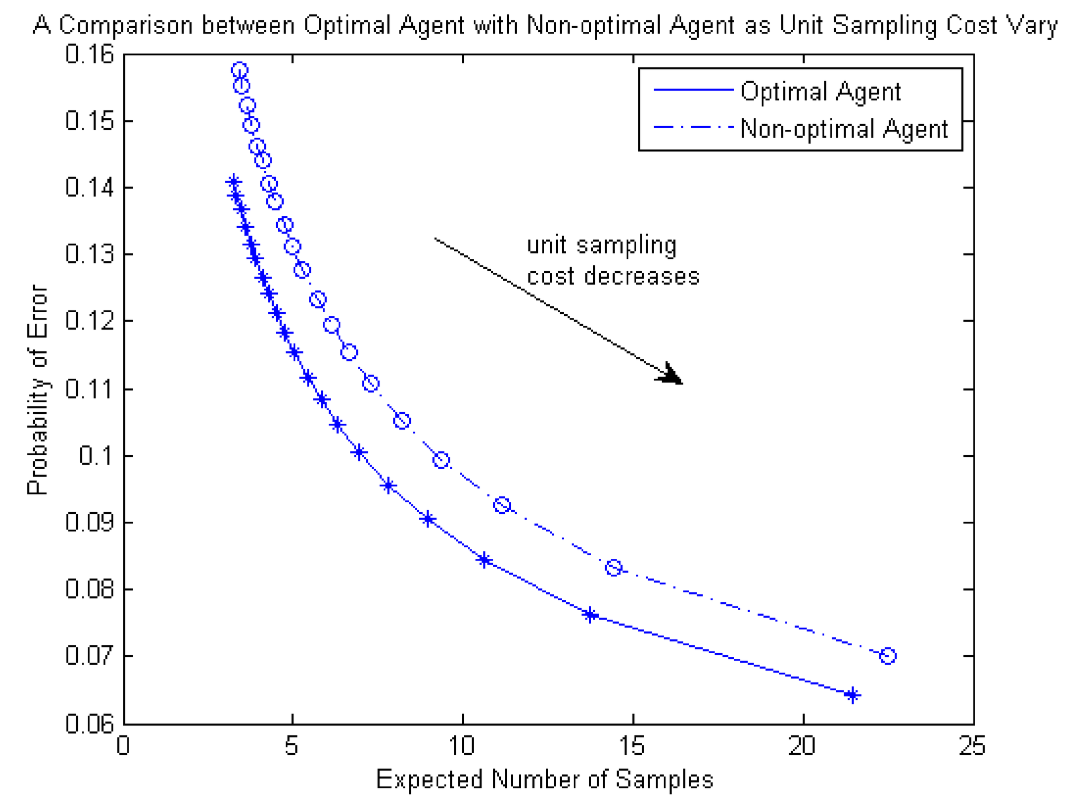

Figure 4: Optimal Agent Compared with Continuous Case

Conclusions

We studied the case where memory size constraint is given to the sequential test. We formulate the problem as an optimal prior probability quantizer design under the sequential setting. Our result shows that the problem can be solved using dynamic programming by formulating the expected cost to continue and the expected cost to terminate as functions of the decision rules, in which setup the quantized prior probability for one hypothesis given the test is in a particular state can be proved to be equal to the proportion of number visiting that state given that hypothesis out of that of all possible states'.

References

[1] D. Teng and E. Ertin, "Optimal quantization of likelihood for low complexity sequential testing", *Proceedings of the IEEE Global Conference on Signal and Information Processing (to appear)*, Dec. 2013.