# Learning to Aggregate Information for Sequential Inferences

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# Problem Statement

## Objective

Learn a mechanism $\{\delta, \gamma, \eta\}$ from training data $\{\mathbf{x}_1^{(0)}, \mathbf{x}_2^{(0)}, \cdots, \mathbf{x}_M^{(0)}\}$ and $\{\mathbf{x}_1^{(1)}, \mathbf{x}_2^{(1)}, \cdots, \mathbf{x}_N^{(1)}\}$ to sequentially classify testing sequence.

$\delta : \mathbb{R} \rightarrow \{0, 1\}$ : stopping rule

$\gamma : \mathbb{R} \rightarrow \{0, 1\}$ : final decision rule

$\eta : \mathbb{R}^d \rightarrow \mathbb{R}$ : information accumulation rule

## Criterion

Minimize $\omega_0 N_0 + \omega_1 N_1$ given $P_D$ and $P_F$ constraints

## Assumption

$\{\mathbf{x}_1^{(0)}, \mathbf{x}_2^{(0)}, \cdots, \mathbf{x}_M^{(0)}\}$ and $\{\mathbf{x}_1^{(1)}, \mathbf{x}_2^{(1)}, \cdots, \mathbf{x}_N^{(1)}\}$ are conditionally iid

# Our Method

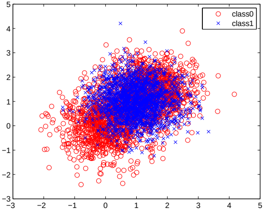
## Optimization problem derived from Martingale theory

$$\hat{r}^{\star} = \arg\min_{\hat{r}} \frac{\omega_0}{\int -\log(\hat{r})p_0} + \frac{\omega_1}{\int \log(\hat{r})p_1}$$
$$\text{s.t.} \quad \int \hat{r}p_0 = 1 \text{ and } \int \hat{r}^{-1}p_1 = 1$$

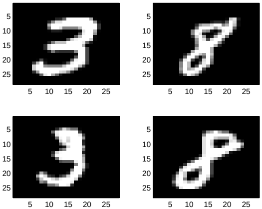
## Optimization problem by imposing Reproducing Kernel structure

$$\hat{r}^{\star} = \min_{\alpha} \frac{\omega_0}{\frac{1}{M}\sum_{j=1}^{M} \alpha^{\mathrm{T}}\mathbf{K}(\mathbf{x}_j^{(0)})} - \frac{\omega_1}{\frac{1}{N}\sum_{i=1}^{N} \alpha^{\mathrm{T}}\mathbf{K}(\mathbf{x}_i^{(1)})}$$
$$\text{s.t.} \quad \frac{1}{M}\sum_{j=1}^{M} \exp(-\alpha^{\mathrm{T}}\mathbf{K}(\mathbf{x}_j^{(0)})) = 1 \text{ and } \frac{1}{N}\sum_{i=1}^{N} \exp(\alpha^{\mathrm{T}}\mathbf{K}(\mathbf{x}_i^{(1)})) = 1$$

# Experimental Results



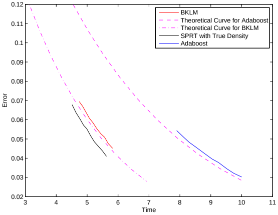

(a) Synthetic 2D Gaussian Mixture



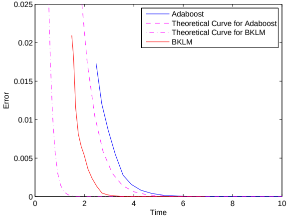
(b) MNIST Handwritten Digits

Figure: Data used in experiments

# Experimental Results




(a) Error vs Expected Sampling Cost for Synthetic Data Example




(b) Error vs Expected Sampling Cost for MNIST Digits Data

Figure: Sequential Classification Experiments