Kronecker PCA Based Space-Time Adaptive Processing (STAP)

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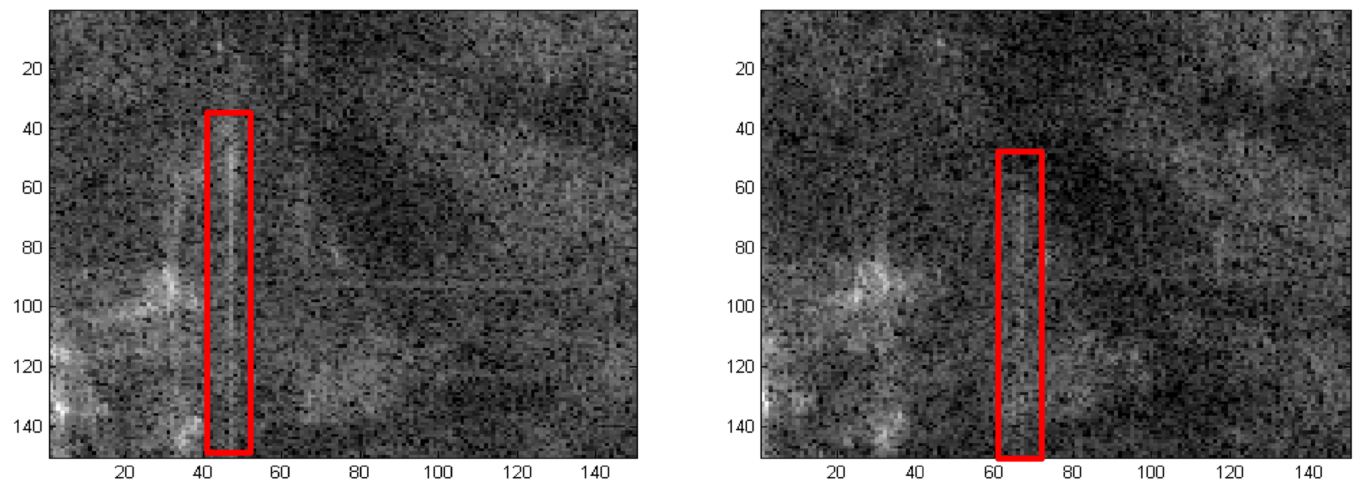
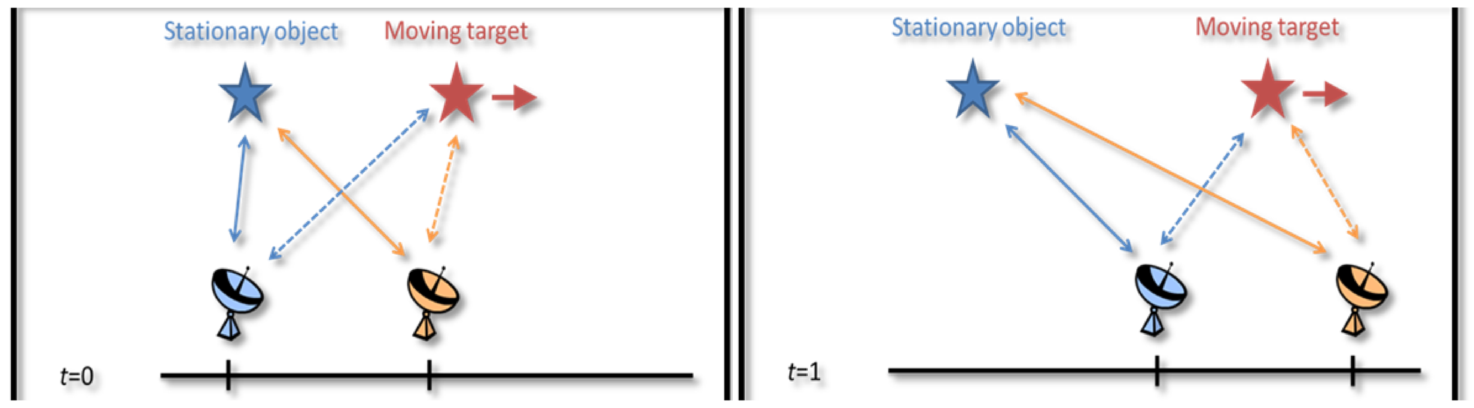
Introduction and Motivation

- Goal: Detect **moving targets** in multiantenna SAR.
  - Synthetic Aperture Radar (SAR): detects stationary targets well, high resolution.
  - Classical GMTI radar: minimum detection speed, low resolution.
  - SAR: **smears** moving targets, hence "buried" in clutter.



- Multiple (p) antennas: moving targets have phase shift.



- Usual multichannel **clutter suppression** methods: ATI, DPCA, STAP
- STAP uses sample covariance to learn clutter distribution
  - p antennas by q pulses/pixels – needs many training samples.
- Kronecker PCA [1]: covariance regularization designed for **dimensional decompositions**.

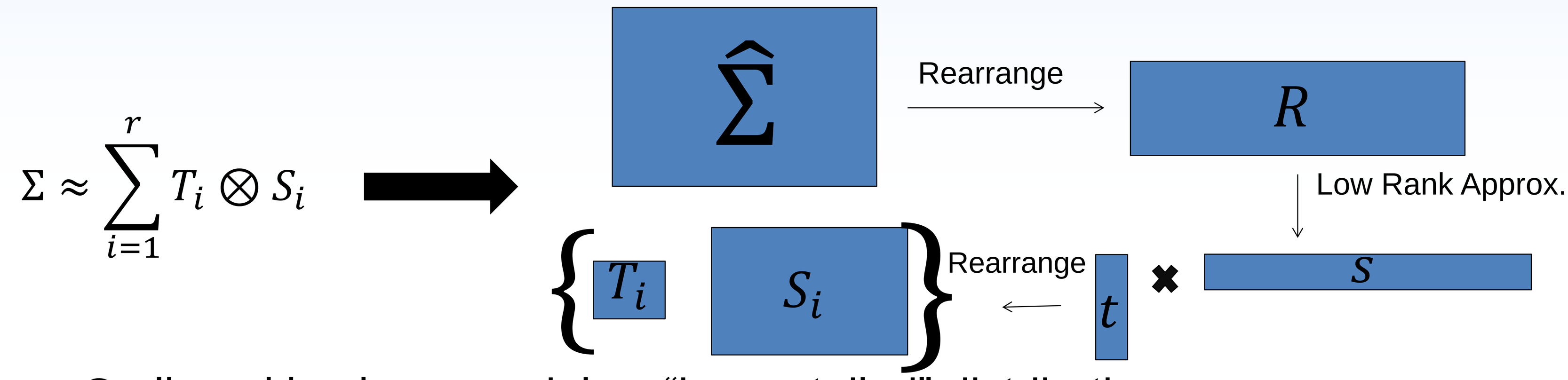
STAP

- Design filter A to cancel clutter ($x = x_{clutter} + x_{noise} + x_{targ}$):
$$y = Ax$$
- Classical oracle canceler
  - Difficult to estimate, typically cancels targets as well.
$$A = \Sigma^{-1} = (E[xx^H])^{-1}$$
- Subspace approach
  - Each antenna receives at the same location at different times.
- Result: clutter (ordered channel vs. pulse) ideally **in 1D subspace**.
$$x = \alpha\beta^T$$
where β (random) depends on random clutter/targets.
- Estimate α from example range bins
  - Standard STAP [2]: estimate by projecting onto r principal components (u_i) of sample covariance (SCM):
$$A = I - \sum_{i=1}^{r} u_i u_i^T$$
  - **Our approach**: Clutter + noise covariance should be (ρ models additive noise, τ random texture):
$$\Sigma = E[\tau] T \otimes S + \rho I, \qquad T = \alpha\alpha^T$$
- This gives possible subspace projection filters:
$$A = (I - \alpha\alpha^T) \otimes I, \text{ and } A = (I - \alpha\alpha^T) \otimes S^{-1}$$

This research was partially supported by the ARO MURI grant ARO W911NF-11-1-0391 and by the AFRL grant AFOSR FA8650-07-D-1220-0006.
PA Approval #88ABW-2014-5281.

KronPCA Covariance Estimation

- Rearrangement operator $\mathcal{R}(\cdot)$: converts Σ to low rank matrix.



- Outlier mitigation – model as "heavy tailed" distribution.
  - Allows **moving targets to be included in training**.
- Shrunk Maximum Likelihood iterative method (elliptical distribution):

Algorithm 1 Robust DC-KronPCA Shrinkage Estimation

```
1:  s_i = x_i / ||x_i||_2, ∀i.
2:  Run Chen iterations until convergence, obtain Σ̂.
3:  Σ̃ ← Σ̂.
4:  while not converged do
5:      T̂ = KronPCA_T{Σ̃}.
6:      while not converged do
7:          Run Tyler iteration: Σ̃ ← (pq/n) Σ_{i=1}^{n} s_i s_i^T / (s_i^T Σ̂^{-1} s_i)
8:          Ŝ = (1/T) Σ_{i,j=1}^{q} [T̂^{-1}]_{ij} Σ̃(j,i).
9:          Σ̂ ← T̂ ⊗ Ŝ
10:         Σ̂ ← (1 − ρ) (pq / trace(Σ̂)) Σ̂ + ρI
11:     end while
12: end while
13: Return Σ̂.
```

- Simplification for **reduced complexity** and generalizability to multiple Kronecker terms [1,3]: LS objective, provably **consistent**.
$$\hat{T} = \mathrm{vec}^{-1}\{\sqrt{\sigma_1} u_1\}, \qquad \hat{S} = \mathrm{vec}^{-1}\{\sqrt{\sigma_1} v_1\}$$
$$\mathcal{R}(\Sigma_{SCM}) = U \mathrm{diag}(\sigma) V^T, \qquad \Sigma_{SCM} = \frac{1}{n}\sum_{i=1}^{n} s_i s_i^T$$

Performance Analysis

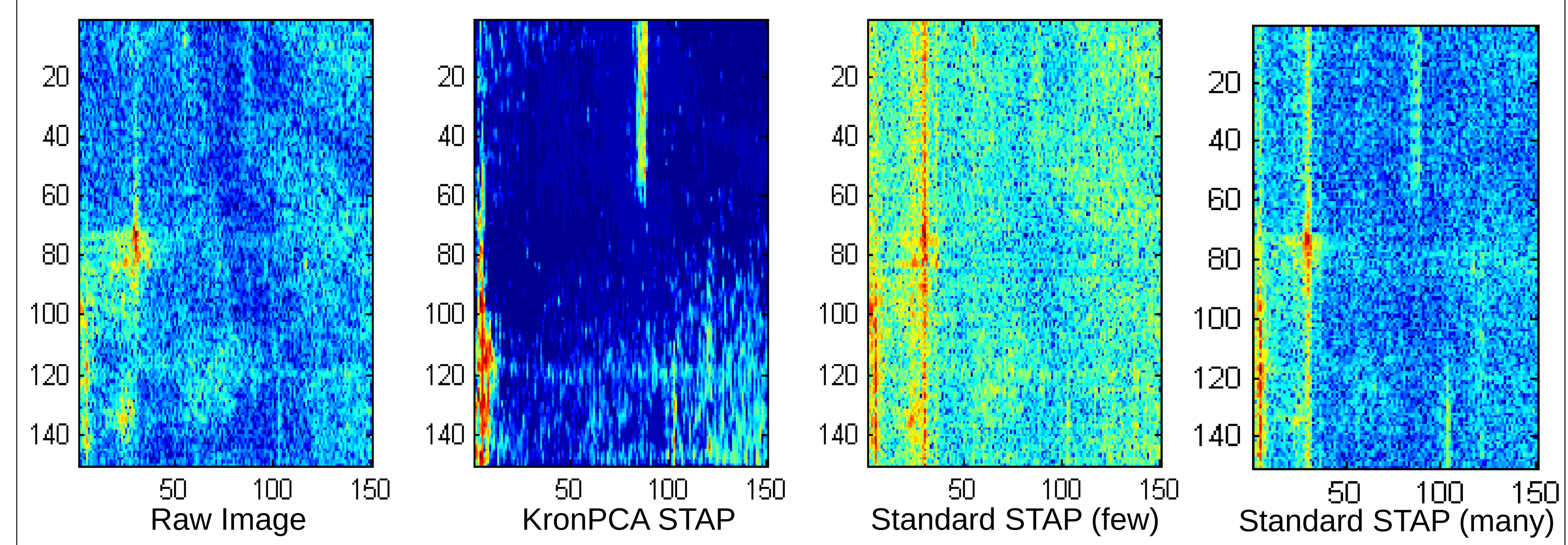
- Parameter reduction – fewer training samples. Model clutter as SIRV [2].
- Define λ to be the SINR loss (SINR relative to infinite training samples).
- Standard rank r STAP [2] ($r \approx q$ theoretically, ϕ_i basis coefficients).
$$E[\lambda] = 1 - \frac{1}{n}\sum_{i=1}^{r}\left(\frac{E[\tau]\phi_i + \rho}{E[\tau]\phi_i}\right)^2 \approx 1 - \frac{r}{n}$$
- Generally [2],
$$\lambda = 1 - d^H(\Delta\Sigma)M(\Delta\Sigma)d$$
so for KronPCA $E[\lambda]$ can be found using the CRB of (Werner 2008).
- KronPCA best/worst case: As $n \to \infty$, $E[\lambda] \to \kappa$
$$1 - \frac{1}{n}\left(\frac{E[\tau]\phi_1 + \rho}{E[\tau]\phi_1}\right)^2 \le \kappa \le 1 - \frac{1}{qn}\left(\frac{E[\tau]\phi_1 + \rho}{E[\tau]\phi_1}\right)^2$$
- Computational learning complexity: LS method $< O(p^4 q^2)$ [1]
- Filtering complexity $O(pn)$ where p is the number of antennas.

Discussion

- KronPCA: **separation of spatial and temporal processing**
  - Can **directly compute** amount of **spatial cancelation** via division.
    - Ensures very high amplitude stationary targets not retained.
  - Spatial covariance not low rank and is only related to clutter.
- Multichannel change detection via joint STAP.

Results

- Public release Gotcha GMTI challenge dataset [4].
  - Many moving targets in region with field/roads/buildings.
- Most targets untruthed.
- Used ~1 second integration time. $p = 3$ antenna channels; circular SAR.
- KronPCA STAP with cancelation estimation.
- Standard low rank STAP.
  - Train on current image including targets: "few".
  - Train on last four images with targets removed: "many".

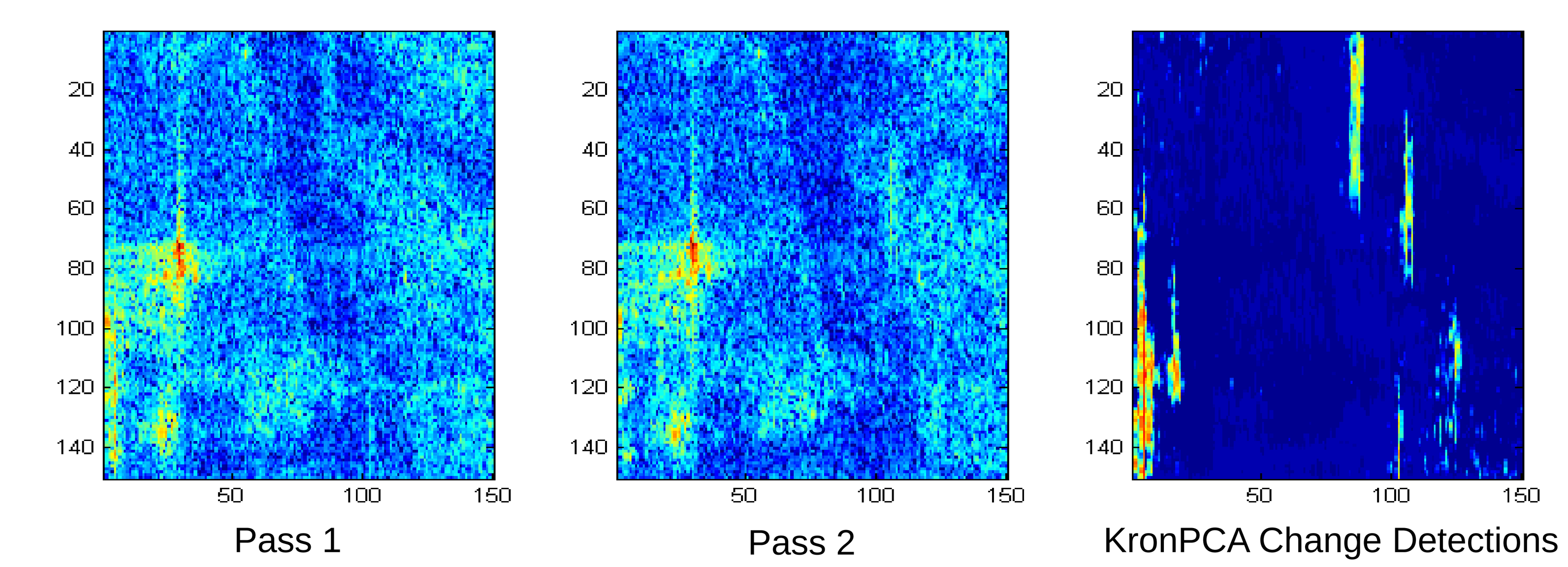


- Example range bin intensity reduction. (KronPCA-T includes temporal and spatial processing.) Note SCM method smears targets.

Method	KronPCA (few)	SCM (few)	KronPCA (many)	KronPCA-T (many)	SCM (many)
Strong Clutter	.04	.17	.04	.04	.04
Weak Target	.5	.5	.6	.8	.2

Clutter vs. Target Cancelation

- Multichannel change detection. Bright spots are detected changes.



Conclusions

- Showed that the clutter covariance ideally has a Kronecker product structure with a rank-one channel component.
- Exploited this in STAP with encouraging results using real data.
- Derived amount of clutter cancelation achieved under SIRV clutter model.

References

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