Value of Information Sharing in Network Signal Detection

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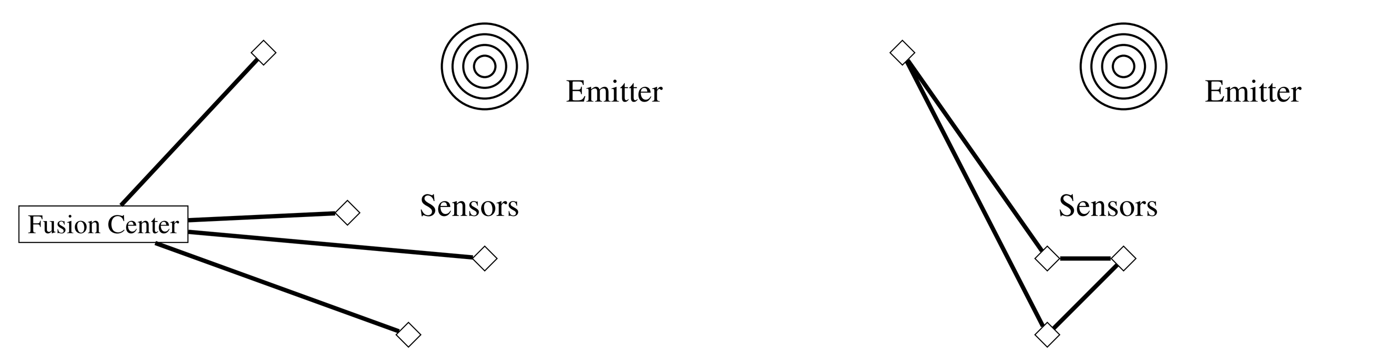
Introduction

- Standard methods for detecting the presence of a common but unknown signal in data collected at multiple geographically distributed sensors rely on total accumulation of the collected data for simultaneous analysis



  - A typical application is detecting the presence of an uncharacterized emitter
  - Processing performed on the aggregated data may be regarded as a data fusion mechanism: it provides a single global answer (i.e., $\to H_1$ / $\to H_0$) from data collected at more than one sensor
- Sensor scenarios that motivate this research involve *incomplete* data accumulation
- For a particular class of multiple-channel detectors, the *value of a link in the network graph* is measured in terms of the difference in detection performance with the link is present between two nodes versus using a maximum-entropy surrogate for data shared between the nodes
- For example, the sensors collecting data may be networked in such a way that complete sharing of data between all pairs of sensors is not viable
- We demonstrate detection performance in such scenarios where maximum-entropy methods are used to "surrogate" information between sensors that may not be in direct communication, providing a mechanism for quantifying the value of information sharing within the network

Generalized Coherence Detection

- Multiple-channel detection using the Generalized Coherence (GC) estimate formed from the channel data is a well-established method [2,3]
- Given $M \geq 2$ complex data channels and finite samples $X_1, \ldots, X_M \in \mathbb{C}^N$ from each channel, the GC estimate obtained from these measurements is

$$\hat{\gamma}^2(X_1, \ldots, X_M) = 1 - \frac{\det G(X_1, \ldots, X_M)}{||X_1||^2 \cdots ||X_M||^2}$$

where $G(X_1, \ldots, X_M)$ is the $M \times M$ Gram matrix

$$G(X_1, \ldots, X_M) = \begin{bmatrix} \langle X_1, X_1 \rangle & \cdots & \langle X_1, X_M \rangle \\ \vdots & \ddots & \vdots \\ \langle X_M, X_1 \rangle & \cdots & \langle X_M, X_M \rangle \end{bmatrix} \qquad (1)$$

- In typical multiple-channel detection applications, $\hat{\gamma}^2$ is compared to a threshold to decide between signal-present (H_1) and signal-absent (H_0) hypotheses
- The statistical properties and performance of the GC estimate as a multi-channel detection statistic have been studied extensively, specifically when all information is shared [1-3,7]

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The Method of Maximum-Entropy

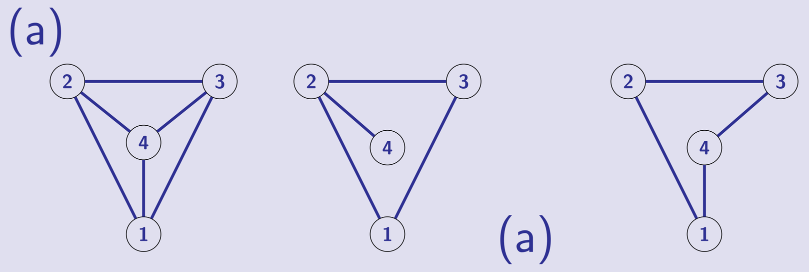
- Given a complex random variable x_k associated with each sensor, modeling data samples collected, the ability to communicate between sensors k and j linked by an edge in the network graph permits estimation of the covariance $\operatorname{cov}(x_k, x_j)$
- In complete sensor network scenarios, it is possible to estimate the full $M \times M$ covariance matrix C
- Assuming the x_k are independent with mean zero, the standard estimate $\hat{C}$ of C is proportional to G in (1)

Computation of Surrogate Values

- The method of maximum-entropy holds that missing values in C should be surrogated to introduce no new assumptions about the nature of the random variables or the network
- The joint distribution of $x_1, \ldots, x_M$ that best describes the current knowledge (i.e., the covariance estimates for pairs of directly connected sensors) with no further assumptions is the maximum entropy distribution constrained by the available data
- The problem of finding the maximum-entropy completion of a covariance matrix has been studied in prior literature, where it is noted that the covariance matrix of this maximum-entropy distribution will have the property that its inverse will have zeros in positions corresponding to the missing covariances [6]

Example

Consider the four-sensor network ($M = 4$) obtained by removing two links from a complete network (a)



(a) (b)

The network topology in (b) gives

$$\hat{C} = \begin{bmatrix} 1 & p & q & \hat{\alpha} \\ p^\dagger & 1 & r & s \\ q^\dagger & r^\dagger & 1 & \hat{\beta} \\ \hat{\alpha}^\dagger & s^\dagger & \hat{\beta}^\dagger & 1 \end{bmatrix}$$

Maximum-entropy surrogate values for $\hat{\alpha}$ and $\hat{\beta}$ are obtained by the noting the 1-4 and 3-4 entry of $\hat{C}^{-1}$ will be zero when $\hat{\alpha}$ and $\hat{\beta}$ assume the desired valued. With the surrogated values, the detector proceeds to apply the generalized coherence detector as though all data were available

References

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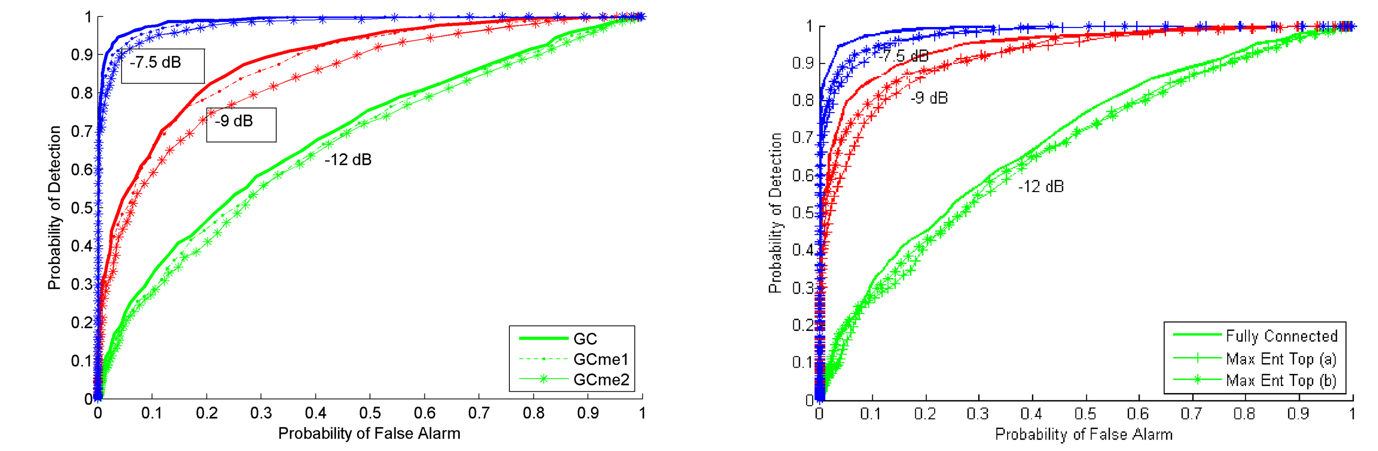
Simulation Results

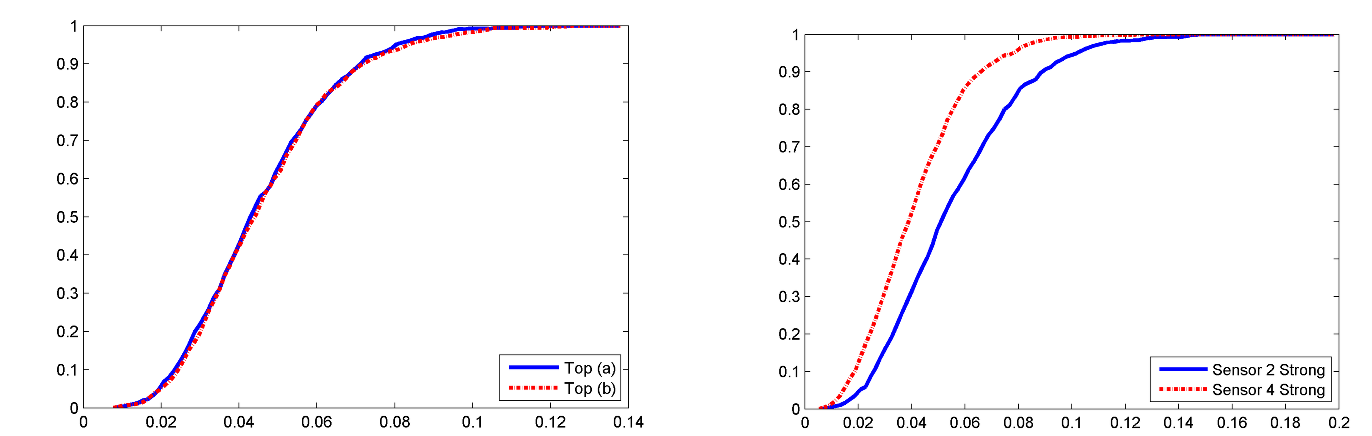
- The effect on detection performance of replacing missing data by maximum-entropy surrogate values has been examined experimentally [4,6]



- A link between sensors is precisely as valuable as the performance gain it enables over the use the maximum-entropy surrogate in place of its datum
- Detection performance for two incompletely connected topologies (each with two surrogated values) suggests that performance may be identical

Null Distribution of Surrogated Coherence

- Knowledge of the distribution of the data under the H_0 assumption that the x_k are independent zero-mean white complex gaussian noise vectors ensures that detection thresholds that correspond to desired false alarm probabilities can be analytically determined
- Without surrogation, conditional distributions under the signal-present and signal-absent hypotheses have been studied [1,3]
- Recording the distribution of the surrogated $\hat{\gamma}^2$ given H_0 with 1,000 degrees of freedom provides empirical cumulative distribution functions for the two distinct four-sensor network topologies (left)



- By standard goodness of fit tests, the distributions under H_0 are indistinguishable
- The invariance to network topology is, however, not expected to hold under the alternative H_1 hypothesis, particularly when channels carry signal of differing strengths (right)

Discussion

- Further empirical tests for various network sizes and topologies show that different network topologies on M sensors with equal numbers of surrogations give indistinguishable distributions of the generalized coherence estimate under H_0, hence:
- **Conjecture**: The null distribution of the surrogated coherence estimate does not depend on the network topology, only on the number of links present.
- If this holds, it will only be necessary for the fusion center to store one set of detection thresholds for each number of possible surrogated values rather than one set for each possible network topology