Learning Latent Variable Gaussian Graphical Models

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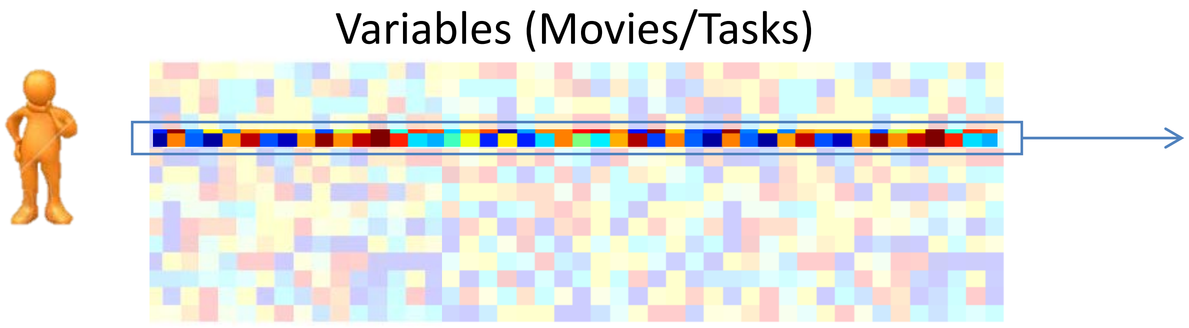
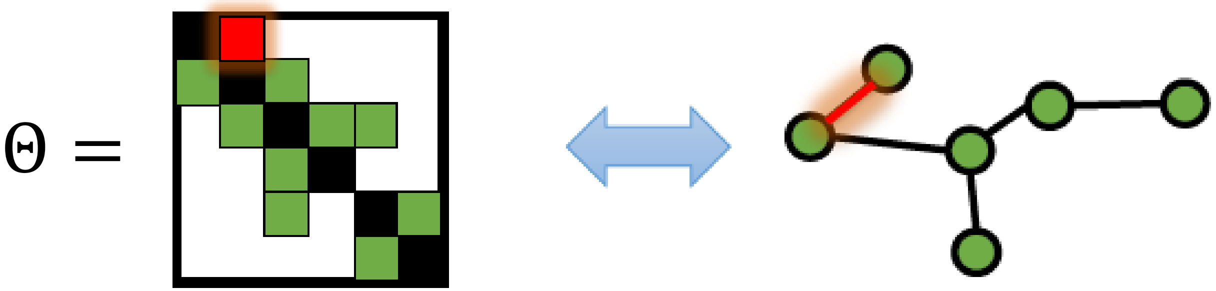
Reference: Z. Meng, B. Eriksson, A. Hero, "Learning Latent Variable Gaussian Graphical Models," International Conference on Machine Learning (ICML), Beijing 2014

Motivation: Recommender Systems

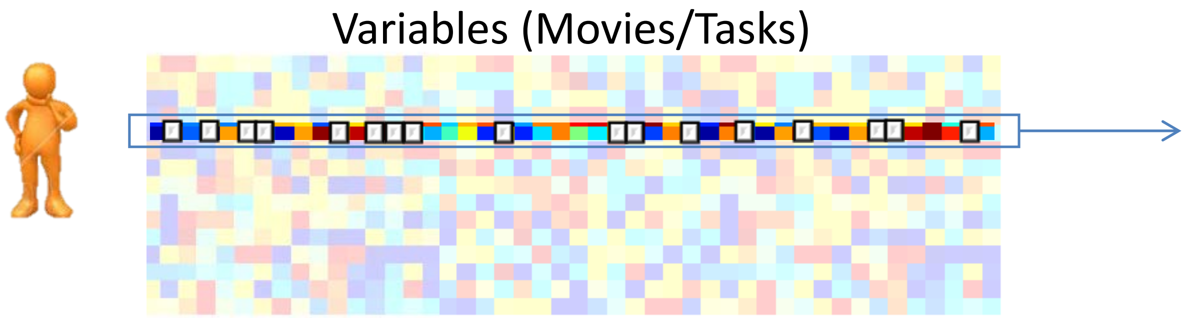
Fully observed matrix $x \sim N(0, \Theta^{-1})$.



Extract sparse precision matrix.



Partially observed random matrix.

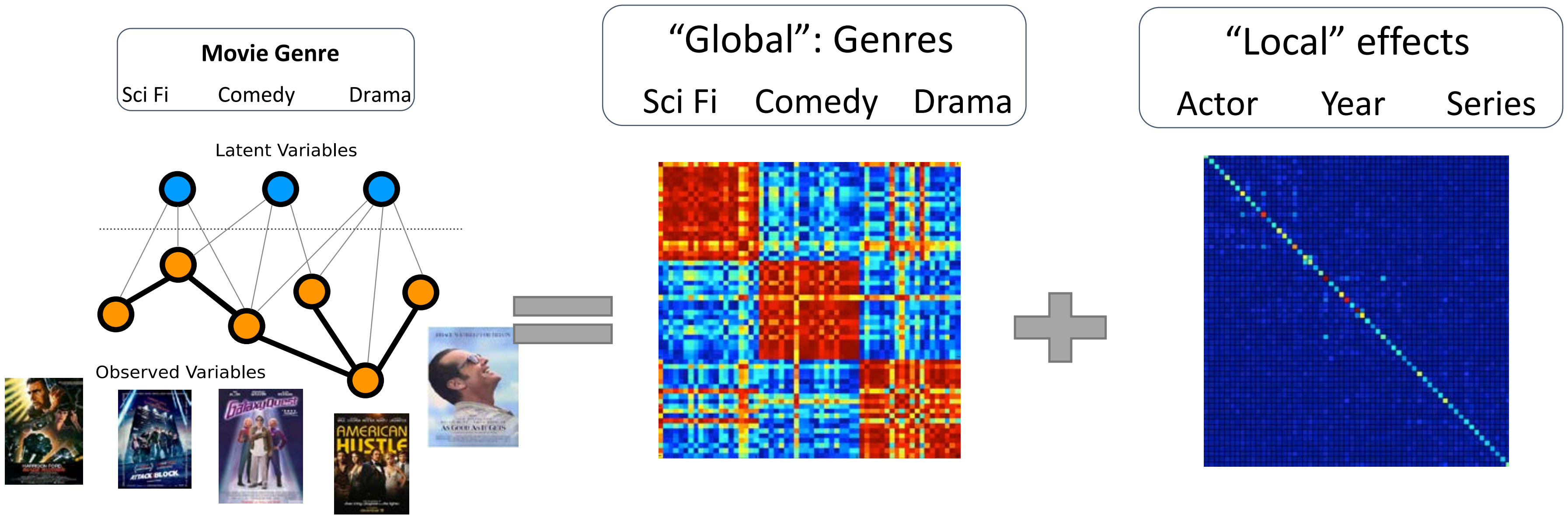


Use CME matrix completion rule.

$$x_{\text{miss}} \mid x_{\text{obs}} \sim N(-\Theta_{m,m}^{-1}\Theta_{m,o}r_o, \Theta_{m,m}^{-1})$$

$$\widehat{x}_{\text{miss}} = -\Theta_{m,m}^{-1}\Theta_{m,o}r_o$$

Latent Variable GGM (LVGGM)



Regularized maximum likelihood estimator of sparse+low rank covariance matrix:

$$\min_{\mathbf{S},\mathbf{L}} \ \langle \widehat{\boldsymbol{\Sigma}}, \mathbf{S}+\mathbf{L} \rangle - \log\det(\mathbf{S}+\mathbf{L}) + \lambda\|\mathbf{S}\|_1 + \mu\|\mathbf{L}\|_*$$

$$\text{s.t.} \quad -\mathbf{L} \succeq \mathbf{0}, \ \ \mathbf{S}+\mathbf{L} \succeq \mathbf{0},$$

Chandrasekaran, Parrilo, and Willsky, Latent Variable Graphical Model Selection via Convex Optimization, Annals of Statistics,Vol. 40, No. 4,2012.

Our contribution: high-dimensional learning rates for precision matrix estimator:

$$\|\widehat{\Theta} - \Theta^*\|_F \leq \tilde{c}_1 \sqrt{\frac{s \log p}{n}} + \tilde{c}_2 \sqrt{\frac{r_{\text{eff}} \cdot r \log(2p)}{n}}.$$