Robust Surface Reconstruction

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Motivation
As datasets grow in size but loose in accuracy
- models should be robust to noise and outliers
- representations efficient
- algorithms scalable and parallelizable

Formulation
Input: oriented point cloud \( \{x_i, n_i\}_{i=1}^{n_p} \)
Output: indicator function of the surface

topology free: \( S = \{x: \chi(x) = 0\} \)

Key: \( \nabla \chi(x_i) = n_i \)

State of the art: least squares
[Kazhdan13, Calakli11] are least-squares models
Optimization reduces to a linear system

Our method: robust and efficient
Robust Model:
\[
\min_{x} \alpha \sum_{k=1}^{n_p} f(\chi(x_k)) + \beta \sum_{k=1}^{n_p} f(\nabla \chi(x_k) - n_k) + \frac{\gamma}{2} \int_{\Omega} w ||\nabla \chi||_{F}^{2}
\]

Efficient Discretization: Hierarchical B-Spline Basis
\( \chi \) is a linear combination
\[
\chi(x) = \sum_{i=1}^{n} c_i \phi_i(x) = \Phi(x)c
\]
\( c \in \mathbb{R}^n \) coefficients, unknown
\( \phi_1, \ldots, \phi_n \) adaptive piecewise-polynomial basis

Efficient Convex optimization: not as easy as a linear system
- discretize the integral with quadrature rules
\[
\min_{x} \alpha \sum_{k=1}^{n_p} f(\chi(x_k)) + \beta \sum_{k=1}^{n_p} f(\nabla \chi(x_k) - n_k) + \frac{\gamma}{2} \sum_{i=1}^{n} w_i ||\nabla \chi(q_i)||_{F}^{2}
\]

Why it is robust: data terms
\[
f(v) = \begin{cases} \frac{1}{2} ||v||_{2}^{2}, & ||v||_{2} < \epsilon \ \\ \epsilon (||v||_{2} - \frac{1}{2}) ||v||_{2}^{2}, & ||v||_{2} \geq \epsilon \end{cases}
\]
Avoid the shrinking bias of least squares

Why it is robust: regularizer
\[
\min_{x} \ldots \int_{\Omega} ||\nabla \chi||_{F}^{2} = \min_{x} \ldots \int_{\Omega} ||Dn||_{F}^{2}
\]
Allows sharp changes in orientation

Why it is efficient: 1st-order alg.
Initialize variables to zero, St. \( r, \sigma, \beta > 0 \)
while \( ||x^{n+1} - x^n|| \geq \epsilon \) do
\[
\begin{align*}
\gamma^{n+1} & = \alpha \min(1, \frac{\|w_i\|_2}{\|w_i\|_1}) \text{sign}(w_i) \\
\delta & = \zeta + \sigma \beta (x^n - x^{n-1}) \\
\lambda & = \zeta + \sigma \beta (x^n - x^{n-1} - n_k) \\
\rho^{n+1} & = \beta - \zeta + \sigma \beta (x^n - x^{n-1} - n_k) \\
\rho^{n+1} & = \beta - \zeta + \sigma \beta (x^n - x^{n-1} - n_k) \\
\end{align*}
\]
end