Optimal measurement matrix design for structured signals in clutter and noise

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Problem statement

$$Y = \Phi(X + C) + W$$

- Signal $X \sim \mathcal{N}(0, \Sigma_X) \in \mathbb{R}^N$ and Clutter $C \sim \mathcal{N}(0, \Sigma_C) \in \mathbb{R}^N$,
- Σ_X of rank $M < N$
- Compressive regime $\Phi \in \mathbb{R}^{K \times N}$, $M \leq K < N$,
- Objective:

$$\min_{\Phi} Tr \left[\Sigma_X \left(I + \frac{\Sigma_X^{1/2} \Phi^T}{\sigma_n^2} \left(\frac{\Phi \Sigma_C \Phi^T}{\sigma_n^2} + I \right)^{-1} \Phi \Sigma_X^{1/2} \right)^{-1} \right] \quad (1)$$

Subject to $Tr\left(\Phi\Phi^T\right) \leq P.$

Prior Work

Renna, Calderbank, Carin solved the problem for the Clutter-free case using the majorization theory tools developed in Palomar and derived a water-filling type solution for the design of the sensing matrix.

Our Approach

1. Apply a whitening filter to transform the combination of projected clutter and noise

$$Z = C_\Phi \Phi X + C_\Phi \left(\Phi C + W \right) = AX + N \tag{2}$$

$$\text{Where, } C_\Phi = \sigma_n (\Phi \Sigma_C \Phi^T + \sigma_n^2 I)^{-1/2}.$$

2. Solve the whitened problem but enforcing the constraint on Φ.
3. Map the solution A^* back to a corresponding Φ^* under a particular condition that $I - \frac{A^* \Sigma_C {A^*}^T}{\sigma_n^2}$ is positive definite.

$$\Phi^* = (I - \frac{A^* \Sigma_C {A^*}^T}{\sigma_n^2})^{-1/2} A, \tag{3}$$

Phase transition

We derive constraints on the geometry of the signal and clutter subspaces for the reconstruction error to vanish as noise variance reduces to 0.

- The number of measurements $K \geq r_X$, where $r_X = rank(\Sigma_X)$.
- $r_X + r_C \leq N$, where $r_C = rank(\Sigma_C)$.
- None of the Principal eigen vecotrs of projected signal should be in the span of the Principal eigen vectors of the projected clutter.