A Novel Framework for Decentralized Supervisory Control with Communication

George Barrett
grbarret@eecs.umich.edu

Stéphane LaForteune
stephane@eecs.umich.edu

Department of Electrical Engineering and Computer Science
The University of Michigan
1301 Beal Avenue
Ann Arbor, MI 48109-2122

ABSTRACT

The decentralized control problem that we address in this paper is that of several communicating supervisory controllers, each with different information, working in concert to exactly achieve a given legal sublanguage of the uncontrolled system’s model. We present a novel information structure formalism for dealing with this class of problems. Preliminary results are presented which elucidate a fundamental concept in decentralized control problems: the importance of controllers anticipating future possible communications.

1. INTRODUCTION

The decentralized nature of information in many large-scale systems as well as practicality dictate the need for control systems that are also decentralized. Decentralized supervisory control in the absence of communication has been well studied in previous work and many classifications of language structure admitting decentralized control without communication exist including co-observability, decomposability, and strong decomposability.

Control of logical discrete-event systems with communication has been pondered recently. The fundamental alteration to the structure of the control systems is that the controllers observe events generated by the system and are allowed to pass messages in order to attempt to resolve ambiguities and determine correct control actions.

Prior work has yet to introduce a framework that is general enough to address certain fundamental questions concerning decentralized supervisory control with communication. For example:

1. Who should know what and when?
2. Who should communicate with whom?
3. When should controllers communicate?
4. What should controllers communicate?

It is the intention of this paper to present a framework that is general enough to address these questions. The decentralized control problem that we address in this paper is that of several communicating supervisory controllers each with different information, working in concert to exactly achieve a given legal sublanguage of the uncontrolled system’s model. The contributions of this work are:

1. A novel information structure formalism is presented for dealing with this class of problems. The information structure based on extended trace models explicitly characterizes observable actions of all controllers which controllers communicate to other controllers what symbols are communicated when controllers initiate communication and what information may be inferred by each of the controllers following any sequence of actions. Constraining specific components of the information structure yields several classes of solutions.
2. Necessary and sufficient conditions are given for the existence (under certain assumptions) of solutions to the described decentralized supervisory control problem with communication.
3. Necessary and sufficient conditions are given that characterize the class of languages achievable by communicating supervisory controllers that do not anticipate future communications.
4. By comparing the classes of languages described by the previous two contributions we elucidate the significance of controllers that anticipate future communications in decentralized supervisory control problems.

The remainder of this paper is organized as follows. The general modeling framework is presented in Section 2. Existence results for the two cases when controllers do and do not anticipate future communications are presented in Section 3 along with a comparison of the classes of languages achievable in the two cases. The paper concludes with Section 4 where the ideas of this paper are summarized and some future areas for investigation are given.

2. GENERAL FRAMEWORK

When designing communicating supervisory controllers that cooperate to achieve a desired legal behavior the three roles of each controller must be considered: estimation, control, and communication. In general these
three roles cannot be separated and any synthesis procedure must take all into account simultaneously. This section presents a general framework for addressing analysis and synthesis problems in decentralized supervisory control with communication.

Consider a discrete-event system modeled by an automaton $G$ with associated language $L(G)$. Associated with the system is a set of events that can be disabled, $\Sigma_d$, a set of observable events, $\Sigma_o$. The set of all events is denoted by $\Sigma$. The sets of uncontrollable and unobservable events are denoted by $\Sigma_u = \Sigma \setminus \Sigma_d$ and $\Sigma_u = \Sigma \setminus \Sigma_o$ respectively. To control the system, there is a finite set of coordinating controllers represented by $Z = \{1, 2, \ldots, n\}$. Each controller $i \in Z$ has an associated set of events $\Sigma_{a,i} \subseteq \Sigma$ that it can disable. A set $\Xi$ of symbols that is allowed to communicate to other controllers and a set of events $\Sigma_{b,i} \subseteq \Sigma_o$ that it can directly observe. The events that are unobservable to each controller are given by $\Sigma_{u,i} = \Sigma \setminus \Sigma_{a,i}$. To represent the fact that controllers have only partial observations of traces in $L(G)$ a projection operator $P_{\Sigma_{a,i}} : \Sigma^* \rightarrow \Sigma_{a,i}^*$ is defined as follows: $P_{\Sigma_{a,i}}(\sigma) = \sigma$ if $\sigma \in \Sigma_{a,i}$, otherwise $P_{\Sigma_{a,i}}(\sigma) = \epsilon$ and $(\forall \sigma \in \Sigma) P_{\Sigma_{a,i}}(\sigma) = P_{\Sigma_{a,i}}(s)P_{\Sigma_{a,i}}(\sigma)$. The subscript alphabet is not given. The projection then is assumed the subscript is the set of all observable events $\Sigma_o$. The inverse projection of $P_{\Sigma_{a,i}}$ is the mapping $P_{\Sigma_{a,i}}^{-1} : \Sigma_{a,i}^* \rightarrow 2^{\Sigma^*}$ defined as $P_{\Sigma_{a,i}}^{-1}(t) = \{ s \in \Sigma^* \mid P_{\Sigma_{a,i}}(s) = t \}$. We will denote the closed-loop language of the plant $G$ under control of the set of supervising $\Sigma^*$. The trajectories observed by controllers are given by $\text{controller}_{i}$. For making a decision is represented by the unprocessed data available to a controller $i \in Z$ for making a decision is represented by an extended trace $[1]$ of the form:

$$t_i = \sigma_i [\prod_{j \in Z} \Xi_{1,j}^{i,j}] \sigma_{i,2} [\prod_{j \in Z} \Xi_{2,j}^{i,j}] \ldots \sigma_{i,k} [\prod_{j \in Z} \Xi_{k,j}^{i,j}] \in T^i$$

where $\sigma_i \in \Sigma_{a,i} \cup \{\epsilon\}$ (denotes the empty trace) $\Xi_{l,j}$ is a set of symbols communicated from controller $j$ to Controller $i$ and $s \in P_{\Sigma_{a,i}}^{-1}(\sigma_i)$. Note that Controller $i$ only “observes” an $i$-transition if two communications arrive at controller $i$ with no event in $\Sigma_{a,i}$ occurring between them. It will be understood that if $\Xi_{l,j}$ is empty then no communication occurred from $j$ to $i$. We will denote the set of all possible symbols that can be communicated as $\Xi = \cup_{i \in Z} \Sigma_{2,i}$ and the extended traces will be referred to as observation/communication trajectories of $G$’s simply $\pi_{i}$. The trajectories observed by controllers represent the controllers’ information states and are projected versions of the global trajectories. The set of all global system trajectories is

$$T \subseteq \left( \Sigma \left[ \Xi_{n \times n}^{n \times n} \right] \right)^*$$

where $\Xi_{n \times n}$ is the set of all $n \times n$-matrices that have elements that are sets $\Xi_{l,j}$ of communication symbols. A matrix $A_{\Xi} \in \Xi_{n \times n}$ is a communication matrix

with one axis indicating which controller is sending a set of symbols and the other axis indicating receivers. The $j,i$-th element of $A_{\Xi} \subseteq \Xi_{2,i}$ is the set of symbols $\Xi_{l,j}$ being communicated from controller $j$ to controller $i$. The dynamic configuration of the communication channels as captured in this trajectory model is closely related to mobility in Milner’s $\pi$-calculus [7,8].

To derive the trajectory sets $T$, $T \subseteq \gamma$ from the global set of trajectories $\Gamma$ TTA prefix-preserving projection operator $\pi_{i} : T \rightarrow T^i$ is defined as:

$$\pi_{i}(\sigma \circ C_{\Xi}^i) = \left\{ \begin{array}{ll} \epsilon \left[ \prod_{j \in Z} \Xi_{1,i}^{j,i} \right] & \text{if } \sigma \notin \Sigma_{a,i} \text{ and } \exists j \text{ s.t. } C_{\Xi}^j \neq \emptyset, \\
\epsilon & \text{if } \sigma \notin \Sigma_{a,i} \text{ and } \exists j \text{ s.t. } C_{\Xi}^j = \emptyset, \\
\sigma \left[ \prod_{j \in Z} \Xi_{2,i}^{j,i} \right] & \text{if } \sigma \in \Sigma_{a,i} \text{ and } \exists j \text{ s.t. } C_{\Xi}^j \neq \emptyset, \\
\sigma & \text{if } \sigma \in \Sigma_{a,i} \text{ and } \exists j \text{ s.t. } C_{\Xi}^j = \emptyset. \end{array} \right.$$ 

$$\pi_{i}(t \circ C_{\Xi}^i) = \pi_{i}(t) \pi_{i}(\sigma \circ C_{\Xi}^i).$$

Given the observation of a trajectory $t \in T^i$ by controller $i$, the inference map $\Psi_{i}$ reflects the “reasoning” capabilities of controller $i$ given the “knowledge” of the global system behavior, including the “reasoning” of other controllers $\gamma$ all control policies $\Gamma$ and all communication policies $\gamma$.

The information structure [12,15,16] presented here for decentralized supervisory control problems is given by

$$\mathcal{I} := \{ T, (\Sigma_{a,i}, \Xi_{i}, \Psi_{i}) : i = 1, \ldots, n \}.$$

Equation (1) indicates the lack of separation of estimation, control, and communication; the design of control and communication policies are dependent on the information structure $\Gamma$ and the information structure is dependent on the policies to be designed. For example the inference map $\Psi_{i}$ which generate an information basis for the control and communication policies will depend on all of $\Gamma_{\text{and}} T$ is built up from individual control and communication decisions resulting from information derived by the controllers’ inference maps. Due to the dependence of the information structure on the policies to be synthesized it is not possible in general to specify the complete information structure $\text{a priori}$ to the synthesis problem $\Gamma_{i.e.,}$ only the form of the information structure

---

1 Anthropomorphic terminology is common; however, a controller need not be aware of the reasoning involved in its policies.
To complete the modeling framework of this section two additional components will be defined: the set of control policies represented by disablement maps and the set of communication policies represented by communication maps. The set of control policies is \( \Gamma = \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_n \} \) where each \( \Gamma_i \) is a disablement map:

\[
\Gamma_i : \rho(\mathcal{L}(Z/G)) \rightarrow \rho(\Sigma_{c,i}).
\]

Finally, the set of communication policies is \( \Theta = \{ \Theta_1, \Theta_2, \ldots, \Theta_n \} \) where each \( \Theta_i \) is a communication map:

\[
\Theta_i : \rho(\mathcal{L}(Z/G)) \rightarrow \bigcap_{i=1}^{n} \rho(\Sigma_i).
\]

The problem investigated in this paper can now be formally stated:

(P) Given a plant \( G \) with generated language \( \mathcal{L}(G) \) a desired behavior modeled by an automaton \( H \) with language \( \mathcal{L}(H) \subseteq \mathcal{L}(G) \) and a set of controllers \( Z = \{ 1, 2, \ldots, n \} \) construct control and communication policies for the controllers \( \Gamma = \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_n \} \) and \( \Theta = \{ \Theta_1, \Theta_2, \ldots, \Theta_n \} \) such that \( \mathcal{L}(Z/G) = \mathcal{L}(H) \).

3. RESULTS

The purpose of this section is to present necessary and sufficient conditions for the existence of information structures that support a solution to Problem (P) as stated in the previous section. The first result is for the case when the information structure is not necessarily constrained to satisfy any particular property and the second result is for the case when the information structure is constrained such that the controllers do not anticipate future communications. The assumptions used in the results that follow are:

1. The plant is modeled by a finite automaton \( G = (X_G, \Sigma_G, \delta_G, \epsilon_G, x_{0G}) \) with associated language \( \mathcal{L}(G) \) and the desired behavior is modeled by a finite automaton \( H = (X_H, \Sigma_H, \delta_H, \epsilon_H, x_{0H}) \) with language \( \mathcal{L}(H) \subseteq \mathcal{L}(G) \).
2. Controllers are synchronized on the initial state of the system.
3. There are no communication delays.
4. There are no communication losses.
5. Collectively the controllers observe all observable events and control all controllable events \( \Gamma \) i.e., \( \bigcup_{j \in Z} \Sigma_{c,j} = \Sigma_c \) and \( \bigcup_{j \in Z} \Sigma_{o,j} = \Sigma_o \).
6. The joint action of the controllers on the system is captured by the union of the sets of disabled events.
7. For the sake of expository simplicity and without loss of generality it will be assumed that there are only two controllers observing and acting on the system.

The first result of this section is formalized by the following theorem on the existence of information structures that support solutions to (P).

**Theorem 3.1 (Existence)** An information structure exists that supports a solution to Problem (P) iff the following two conditions hold:

1. \( \mathcal{L}(H) \) is controllable with respect to \( \mathcal{L}(G) \) and \( \Sigma_{c,i} \).
2. \( \mathcal{L}(H) \) is observable with respect to \( \mathcal{L}(G), \Sigma_c \) and \( \Sigma_o \).

Furthermore, the information structure has a finite representation and the solution to (P) can be obtained by the communication of controller state-estimates.

Theorem 3.1 implies that Problem (P) has a solution iff the desired language \( \mathcal{L}(H) \) can be implemented by a centralized supervisor. The proof of Theorem 3.1 (see [2]) utilizes the construction of a finite estimator structure and inference maps that utilize the existence of future communications from other controllers along certain trajectories in the system. Memory of prior communications is captured by the state-update rule for the estimator structure. It will be shown that what is important about this result is the fact that the controllers anticipate communication that is not only do these controllers base their “state estimates” on every event they observe and every communication they receive but they also utilize the fact that they can expect future communications from other controllers if certain events occur. The next result provides a characterization of the languages achievable by controllers that do not anticipate future communications that is the controllers’ estimates do not take into account future communications.

Consider two controllers that maintain trace estimates (not state estimates). That is if following a trajectory \( t \in T \) each controller \( i \in Z \) has an estimate \( \psi_i(\tau(t)) \) of trajectories that Controller \( i \) infers could have occurred given its observations of events and communications received. We will say that Controller \( i \) is myopic if \( \psi_i \) does not take into account future communications between the controllers that is if

\[
(\forall t \in T^i) \quad \psi_i(t) = \psi_i(t) \cap \mathcal{L}(H).
\]

Communication between controllers is called two-way if whenever Controller \( i \) sends a message to Controller \( j \) Controller \( j \) responds by sending a message to Controller \( i \). Then we have the following theorem.

**Theorem 3.2** Let \( Z = \{ 1, 2 \} \) be a set of myopic controllers that maintain trace estimates and communicate
their trace estimates two-way following any event observed by either controller. Then Problem (P) can be solved with these controllers iff the following two conditions hold:
1. \( \mathcal{L}(H) \) is controllable with respect to \( \mathcal{L}(G) \) and \( \Sigma_{ci} \):
\[
[s \sigma \notin \mathcal{L}(H)] \land [s \sigma \in \mathcal{L}(G)] \Rightarrow
(\exists i \in Z) [P^{-1}(P(s)) \Sigma_{ci} \sigma \cap \mathcal{L}(H) = \emptyset] \land [\sigma \in \Sigma_{ci}].
\]
It is interesting to compare the class of languages achievable by myopic controllers to the class of languages achievable by controllers that anticipate future communications. It can be shown that the observability condition in Theorem 3.1 can be rewritten as (\( \forall s \in \mathcal{L}(H) \)) (\( \forall \sigma \in \Sigma_{ci} \)):
\[
[s \sigma \notin \mathcal{L}(H)] \land [s \sigma \in \mathcal{L}(G)] \Rightarrow
(\exists i \in Z) [P^{-1}(P(s)) \sigma \cap \mathcal{L}(H) = \emptyset] \land [\sigma \in \Sigma_{ci}].
\]
Comparing this with the second condition in Theorem 3.2 reveals that myopic controllers with arbitrarily large memory and communication resources (i.e., maintaining and communicating arbitrarily large trace estimates) are outperformed by controllers that maintain and communicate finite state estimates but anticipate future communications.

4. CONCLUSION
In this paper a novel framework is given for analysis and synthesis issues in decentralized supervisory control with communication. We also characterize the classes of languages achievable for the two cases of when controllers do and do not anticipate future communications. Anticipation of future communications is described as a condition on the \( \Psi_i \), components of the information structure in the general model. Comparing the two classes of achievable languages reveals the fact that communicating controllers with finite memory and communication resources that anticipate future communications outperform controllers with unbounded memory and communication resources that do not anticipate future communications.
Future challenges and areas of investigation include the development of algorithms to synthesize control and communication policies for controllers that anticipate communications. Extensions to include versions of Problem (P) that allow \textit{tolerance}, i.e., \( L_1 \subseteq \mathcal{L}(Z/G) \subseteq L_2 \) and the inclusion of the effects of delay and loss in the communication channels.

5. ACKNOWLEDGMENTS
The authors wish to thank Feng Lin, Karen Rudie, Jan van Schuppen, and Demosthenis Tenekezis for discussions on decentralized control with communication.
This research was supported in part by ARO grant DAAH04-96-1-0377.

6. REFERENCES
[4] K. Inan \( \Gamma \) \textit{An algebraic approach to supervisory control} \( \Gamma \) Mathematics of Control \( \Gamma \) Signals \( \Gamma \) and Systems 5 (1992) \( \Gamma \) no. 27153–164.
[7] R. Milner \( \Gamma \) \textit{A calculus of mobile processes}, \( \Gamma \) Information and Computation 100 (1992) \( \Gamma \) no. 171–177.
[8] \textit{A calculus of mobile processes}, \( \Gamma \) Information and Computation 100 (1992) \( \Gamma \) no. 171–177.
[16] \textit{Equivalent stochastic control problems} \( \Gamma \) Mathematics of Control \( \Gamma \) Signals \( \Gamma \) and Systems \( \Gamma \) vol. 11 Springer–Verlag 1988 Pp. 3–11.