Active Acquisition of Information for Diagnosis of Discrete Event Systems

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Abstract

A method for determining minimum cost observation strategies for failure diagnosis of discrete event systems is proposed. This "active acquisition" of information method is described for both logical and stochastic discrete event systems.

I. INTRODUCTION

Many types of systems, including communication networks, manufacturing processes, and queueing systems, can be modelled using discrete event systems (DES). An important problem in complex systems modelled by DES is the problem of detecting and isolating failure events.

One approach to the problem of failure detection in DES involves verification of the property of diagnosability (for an overview of this approach, see [1]). Roughly speaking, a DES is diagnosable if any failure that occurs can be diagnosed after a finite delay. In recent years, there has been interest in studying diagnosability of stochastic DES as well ([2],[3]).

A problem related to the verification of the diagnosability property is the sensor selection problem for DES ([4], [5], [6]). In the sensor selection problem, the objective is to find the minimal sets of sensors under which diagnosability is preserved when these sensors are activated for the duration of the discrete-event process.

In some situations, finding a solution to a sensor selection problem may not result in a solution that is optimal in a practical sense. For example, in communication networks, the act of sensing an event at a remote location involves using system bandwidth to send the data to a network co-ordinator. If the sensor is wireless, the act of transmitting data involves using some of the small amount of energy available to the sensor. In these situations, we do not purchase a sensor at the start of the process and let it run for the duration; instead we incur a small cost each time the sensor is used.

If our objective is to minimize the total cost incurred by the active use of sensors, then, roughly speaking, our objective is to use the sensors as infrequently as possible, that is, to determine when it is necessary to actively acquire information along each possible system behavior. This is a different objective than that of the standard sensor selection problem, where the goal is to use as few sensors as possible, but to activate them for the duration of the process.

This paper investigates the use of active acquisition of information in the context of DES. Our objective is to minimize the cost of observing a finite-state machine when a cost is paid each time a sensor is activated, while preserving a diagnosability property similar to that of [7].

The distinguishing characteristic between the verification problems, the sensor selection problems, and the active acquisition problem proposed in this paper is the information structure. In verification problems such as [7], the information available to the observer/diagnoser is specified by a fixed projection or observation mask. In sensor selection problems ([4]-[6]), the objective is to select the fixed observation mask that minimizes the cost associated with purchasing sensors...
that are then activated for the duration of the discrete-event process. In the active acquisition of information problem, the observer actively decides which sensors are to be used based on the information that it has already available. A cost is incurred each time a sensor is activated in an attempt to sense the event associated with that sensor. If a sensor is never activated, the system does not incur a cost from that sensor, even if it is available for the observer to use.

Variations on the active acquisition of information approach have been applied to many classes of systems other than DES. For example, problems involving sensors that can be activated or deactivated based on the system behavior have been considered for many different classes of systems, including centralized and decentralized linear stochastic systems (e.g., [8], [9], [10], [11], [12]), communication networks, ([13], [14]) and operations research [15]. In this paper we consider a version of the problem where the decision as to what sensors are activated is made by a centralized diagnoser; a schematic of this diagnoser is shown in Figure 1. In the architecture this paper considers, the diagnoser contains an observer that reads in data from a DES. It then sends the information it has obtained to a policy maker that instantaneously feeds back to the observer the set of events it should next attempt to observe.

This paper is structured as follows. Section 2 provides a description of the DES model under consideration. Section 3 defines the active acquisition of information problem in the context of DES. An appropriate information state is described in Section 4. Section 5 describes the method used to find an optimal observation policy. The results of this method are illustrated in Section 6. A limited lookahead algorithm for computational savings is described in Section 7. The stochastic analogue of the problem is discussed in Section 8.

II. MODELLING FORMALISM

The DES model under consideration is a finite-state machine, or automaton. An automaton $G$ is formally defined as:

$$G = (\Sigma, X, \delta, x_0),$$

where

- $\Sigma$ is a finite set of events
- $X$ is a finite state space
- $\delta : X \times \Sigma \rightarrow X$ is the partial transition function
- $x_0 \in X$ is the initial state

To simplify the development of the active acquisition problem, we make the following assumptions:

(A1) The automaton $G$ is acyclic. Therefore, there exists a constant $T$ that bounds all the traces in the language generated by $G$. Traces that terminate before reaching the bound $T$ can be extended by adding the appropriate number of $\epsilon$ transitions.
(A2) There is a constant amount of time between the occurrence of two successive events.

Assumption (A1) ensures that the worst case observation cost of the system remains finite and forces the existence of a finite horizon $T$. Assumption (A2) simplifies the development of the concepts of information state and $\sigma$-field that will be used to solve the active acquisition problem. We intend to remove assumption (A2) in a future version of this work.

As there is a constant amount of time between events, we define for all $t \leq T$,

$$L_t = \{ s : s \in \mathcal{L}(G) \land \|s\| = t \}$$

$L_t$ is simply the language that can be realized by the automaton at time $t$.

In the active acquisition problem, an event is called observable if there is an available sensor that can detect its occurrence (although at any moment we may choose not to use that particular sensor) and it is called unobservable if there is no such available sensor. Formally, the event set is partitioned as $\Sigma = \Sigma_o \cup \Sigma_{uo}$, where $\Sigma_o$ is the set of observable events and $\Sigma_{uo}$ is the set of unobservable events.

There is a cost $\nu : \Sigma_o \to [0, \infty)$ associated with activating each sensor in order to identify an occurrence of an observable event. If $\nu(\sigma) = 0$, then $\sigma$ is said to be freely observable; the set of all freely observable events is denoted by $\Sigma_{fo}$. Otherwise, $\sigma$ is said to be costly; the set of all costly observable events is denoted by $\Sigma_{co}$. The cost of an observation action $u \in 2^{\Sigma_{co}}$ is simply the sum of the costs of each event observable under that action:

$$c(u) = \sum_{\sigma \in u} \nu(\sigma)$$

The set of failure events to be diagnosed is $\Sigma_f \subseteq \Sigma$. We can assume that $\Sigma_{fo} \cap \Sigma_f = \emptyset$, as it is a trivial problem to diagnose a failure that can be freely observed.

The set of failure events is partitioned into a set of failure types $\Sigma_f = \Sigma_{f_1} \cup \cdots \cup \Sigma_{f_m}$. If a failure event $\sigma_f \in \Sigma_{f_i}$ occurs, this is equivalent to the phrase “a failure of type $F_i$ occurs.”

Our objective is to find an optimal observation policy that diagnoses $\mathcal{L}(G)$ in the sense defined in the next section.

### III. PROBLEM FORMULATION

The active acquisition problem is a problem of optimization; we wish to find a policy that minimizes the observation cost while allowing for the detection of any failures by the time the process terminates.

Since the automaton is $G$ is acyclic, we simply desire that there exists an observation policy so that when the process terminates at time $T$, we can be certain as to whether or not a failure has occurred. To formalize this notion, we require the following definitions.

**Definition 1.** An information state $\pi$ is set of all possible system behaviors in $L_T$ consistent with the observations that the observer has chosen to make.

At time $t$, the information state consists of the traces of length $t$ that agree with the observations made up to that point extended to time $T$ by appending their postlanguages to the end. The information state at time $t$ can be thought of as some sublanguage of $L_T$.

**Definition 2.** An information state $\pi \in 2^{L_T}$ is called certain if, for all failure types $F_i$, either every $s \in \pi$ contains an event in $\Sigma_{f_i}$ or no $s \in \pi$ contains any event in $\Sigma_{f_i}$.

**Definition 3.** A language $\mathcal{L}(G)$ is diagnosed by an observation policy $g$ if, for all $s \in L_T$, the information state reached by implementing $g$ is certain with respect to all types of failures after $s$ has occurred.

**Definition 4.** Let $H$ denote the set of all policies that diagnose $\mathcal{L}(G)$. The language $\mathcal{L}(G)$ is diagnosable if $H$ is non-empty, i.e., if there exists a policy that diagnoses $\mathcal{L}(G)$.  

The problem under consideration is to find an observation policy that diagnoses $L(G)$ at minimal worst-case cost. Define the performance criterion:

$$J(g) = \max_{s \in L_T} \sum_{t=1}^{T} c^g_t(s)$$

where $c^g_t(s)$ denotes the cost of implementing policy $g$ at time $t$ along the trajectory $s$. The performance criterion is thus the maximum total cost of policy $g$ for $t = 0..T$.

The active acquisition of information problem, henceforth also called the active acquisition problem, is defined as follows.

**Problem Q.** Find a policy $g^* \in H$ such that

$$J(g^*) = \inf(J(g) | g \in H)$$

To solve Problem Q, we need to develop a systematic method of handling the information available to the diagnoser as it implements any policy. The technique for systematically treating this information is discussed in the next section.

IV. INFORMATION STATES FOR ACTIVE ACQUISITION

In an observation or diagnosis problem with a fixed set of activated sensors, the information available to the observer is defined using the projection operation [16]. Under assumption (A2), we define the projection for a timed system as $P : \Sigma^* \rightarrow (\Sigma_o \cup \epsilon_o)^*$

$$P(\epsilon) = \epsilon$$

$$P(\sigma) = \begin{cases} 
\sigma & \text{if } \sigma \in \Sigma_o \\
\epsilon_o & \text{otherwise}
\end{cases}$$

$$P(s\sigma) = P(s)P(\sigma)$$

The symbol $\epsilon_o$ is considered to be observable; it indicates that no event in the alphabet $\Sigma$ was observed at a particular time.

Similarly, the inverse projection is defined as

$$P^{-1}_L(s_o) = \{s \in L : P(s) = s_o\}$$

In general, the inverse projection operation does not yield a single trace, but instead a set of traces in $L(G)$.

The projection operator is in itself insufficient to describe the evolution of information in the active acquisition problem. At each stage of the problem, we choose an observation policy that indicates the set of costly observable events whose sensors will be activated, and thus we do not have a projection that is fixed for all time.

The difficulty therefore in the active acquisition problem is the derivation of a systematic method of approaching how information regarding the system behavior evolves as different events are observed at different stages of the system’s evolution. To develop this method, we use a maximal $\sigma$-field approach. This approach was initially proposed in [17], [18] in the context of general informationally decentralized systems and was further used in [19], [20], [21], [22].

The maximal $\sigma$-fields at $t = 0, 1, \ldots, T$ contain all possible information states, regardless of what observation policy is selected. For $0 \leq t \leq T$, define the functions $\chi_t : L_t \rightarrow 2^{L_T}$ as:

$$\chi_t(s_t) = \{s \in L_T : P(s_t) \text{ is a prefix of } P(s)\}$$

Let $R_t$ denote the range of $\chi_t$; $R_t$ is a subset of $2^{L_T}$ that is also a partition of $L_T$. The maximal $\sigma$-field at time $t$ is then defined as:

$$\mathcal{F}_t = \sigma\{\pi : \pi \in R_t\}$$
At time \( t \), the information state \( \pi(t) \) is necessarily an element of \( F_t \).

Each element of the partition of \( L_T \) generating \( F_t \) is the “finest” information available along a particular trace at time \( t \). Traces that have identical projections onto \( \Sigma_o \) up to time \( t \) are part of the same atom of \( F_t \), since the diagnoser is unable to distinguish among such traces under any policy.

From Equations (10) and (11), we conclude that \( F_0 \subseteq F_1 \subseteq \cdots \subseteq F_T \). The sequence of maximal \( \sigma \)-fields is a filtration [23]. As the system evolves, at each time step the observer filters out certain system behaviors that are not consistent with the observations it made (or did not make) under the observation policy. Such successive operations lead to successive refinements of the observer’s information.

Having developed a method to describe the information state, we can now address the question of how to determine the existence of an optimal observation policy and develop a method to find such a policy if it exists.

V. FINDING AN OPTIMAL OBSERVATION POLICY

In this section we first present a criterion for diagnosability that can be used to determine if an optimal observation policy exists. We then present a method that determines a policy which minimizes a worst case observation cost, subject to the constraint that all failures in the system are diagnosed.

A. EXISTENCE OF AN OPTIMAL POLICY

In order for a solution to Problem Q to exist, the set of admissible observation policies \( H \) must be non-empty, i.e., the language \( L(G) \) must be diagnosable. Therefore the condition for existence of a solution to problem Q is simply the condition for diagnosability.

Theorem 1: \( L(G) \) is diagnosable if and only if all elements of the partition of \( L_T \) that generates \( F_T \) are certain.

Proof. (Sufficiency) Suppose each element of the partition of \( L(G) \) that generates \( F_T \) is certain. Let \( g_{\text{max}} \) denote the policy where \( g_t(\pi_t) = \Sigma_{\pi_t} \) for all \( \pi_t \) and all \( t = 0,1,\ldots,T-1 \), i.e., the policy where all costly sensors are always activated. Along any string in \( L_T \), the only strings consistent with the observations made under \( g_{\text{max}} \) have identical projections onto \( \Sigma_o \); therefore, the information state reached along any string \( s \in L_T \) is an element of the partition of \( L_T \) that generates \( F_T \). Since that information state is \( F \)-certain, \( g_{\text{max}} \) diagnoses \( L(G) \).

(Necessity) We prove necessity by proving the contrapositive statement. Suppose that there exists an element of the partition of \( L(G) \) that generates \( F_T \) that is uncertain. Then there exist two traces \( s_1, s_2 \in L_T \) such that \( P(s_1) = P(s_2) \), where \( P \) is the projection of \( \Sigma \) onto \( \Sigma_o \) and \( s_1 + s_2 \) is uncertain.

Select any observation policy \( g \) and consider the information state reached by implementing \( g \) along \( s_1 \). That information state contains both \( s_1 \) and \( s_2 \); therefore it is uncertain. Since \( g \) was arbitrarily chosen, it follows that there is no policy that diagnoses \( L(G) \).

Having demonstrated a criterion for testing the diagnosability of a language, from this point forward we consider only those languages that are diagnosable.

B. ACTIVE ACQUISITION DYNAMIC PROGRAM

To initialize the active acquisition program, we first assign a cost to all elements of the final \( \sigma \)-field \( F_T \). For all information states \( \pi \in F_T \), define

\[
V_T(\pi) = \begin{cases} 
0 & \text{if } \pi \text{ is certain} \\
\infty & \text{otherwise}
\end{cases}
\]
We therefore declare illegal any final information state in which we are uncertain as to whether or not any failure type $\Sigma_f$ has occurred. Problem Q is thus analogous to a supervisory control problem (with perfect observation) where the objective is to prevent the system from reaching certain illegal states; here we wish to prevent the system from reaching certain illegal information states.

The information state transition functions are a set of partial functions, indexed on time, that define how the information state evolves as events are either observed or not observed. Under an observation policy $g := (g_0, g_1, \ldots, g_{T-1})$, the information state transition function is defined for each $t$ as

$$\hat{\delta}_{gt} : F_t \times \Sigma \to F_{t+1}$$

by

$$\hat{\delta}_{gt} (\pi, \sigma) = \begin{cases} \{ s \in \pi : s_{t+1} = \sigma \} & \text{if } \sigma \in \Sigma_{gt(\pi), \text{obs}} \\ \{ s \in \pi : s_{t+1} \in \Sigma_{gt(\pi), \text{unobs}} \} & \text{otherwise} \end{cases}$$

where $s_{t+1}$ denotes the $(t + 1)$st event in the string $s$, $\Sigma_{gt(\pi), \text{obs}}$ denotes the set of events that are observable under the action $g_t(\pi)$, and $\Sigma_{gt(\pi), \text{unobs}}$ denotes the set that is not observable under that action.

To determine an optimal policy, we set up a dynamic program and perform backwards induction starting at time $T - 1$ using the following equation:

$$V_{t-1}(\pi) = \min_{g(\pi) \in \Sigma_{co}} \{ c_{g(\pi)} + \max_{\sigma \in \Sigma} V_t(\hat{\delta}_{g(\pi), t}(\pi, \sigma)) \} \quad \text{for } \pi \in F_{t-1}$$

An optimal action for an information state $\pi$ at time $t - 1$ is an observation action $u$ that minimizes $V_{t-1}(\pi)$. The solution of the dynamic program, defined by Equations (12)-(14), determines an optimal observation policy $g^* := (g^*_1, g^*_2, \ldots, g^*_{T-1})$ and the corresponding optimal cost $J(g^*) = V_0(L_T)$. The optimal cost is the minimum worst case observation cost that diagnoses $\mathcal{L}(G)$.

VI. EXAMPLE

We illustrate the results of the previous section by applying the active acquisition algorithm to the finite-state machine in Figure 2. In this example, $\Sigma_{uo} = \{ \sigma_f, \sigma_{uo} \}$, $\Sigma_{co} = \{ a, b, c \}$, $\Sigma_f = \{ \sigma_f \}$, and $T = 3$.

The longest trace in the language of this automaton contains three events. The final $\sigma$-field is thus $F_3$, defined as:

$$F_3 = \sigma(\sigma_{uo}bb, \sigma_fca, \sigma_{uo}aa, \sigma_fab)$$

The elements of $F_3$ are listed in the first column of Table I. For each $\pi \in F_3$, we assign a cost based on whether or not the information state is certain; these costs are shown in the second column of Table I.
The strings \( t \) four observation actions must be evaluated at the cost of the state necessary to determine an optimal observation policy for all information states. 

Table II indicates that an optimal observation action for this information state is \( \sigma_{ab} \). Since both events are unobservable, the dynamic programming equation indicates that \( V_2(t) = 0 \).

Table II shows an optimal policy \( g^* = (g_0^*, g_1^*, g_2^*) \) for all information states that are reachable under \( g^* \). Note that, in order to determine which information states were reachable, it was necessary to determine an optimal observation policy for all information states.

The \( \sigma \)-field \( F_2 \) is a proper subset of \( F_3 \), given by the following:

\[
F_2 = \sigma(\sigma_{ab}, \sigma_{fca}, \sigma_{aa} + \sigma_{fa})
\]

The strings \( \sigma_{ab} \) and \( \sigma_{fa} \) have an identical projection up to time \( t = 2 \) and thus are part of the same element of the partition of \( L_2 \) that generates \( F_2 \).

For each \( \pi \in F_2 \), the cost \( V_2(\pi) \) is calculated using the dynamic programming equation:

\[
V_2(\pi) = \min_{g(\pi) \in g \Sigma \Sigma} \{c(g(\pi)) + \max_{\pi \in \Sigma} V_3(\hat{g}(\pi)(\pi, \sigma))\}
\]

The determination of an optimal observation action for the information state \( \sigma_{ab} + \sigma_{fa} \) at time \( t = 2 \) is shown in Table II. Since \( c \) cannot be the next event from this information state, four observation actions must be evaluated at \( \sigma_{ab} + \sigma_{fa} \): \( \emptyset \), \( \{a\} \), \( \{b\} \), and \( \{a,b\} \).

Table II indicates that an optimal observation action for this information state is \( \{b\} \); therefore the cost of the state \( V_2 \) is \( \nu_0 = 1 \). The values of \( V_2 \) for all \( \pi \in F_2 \) are shown in Table I.

All strings in \( L_3 \) have the same projection up to \( t = 1 \) and thus \( F_1 = \sigma(\emptyset) = \{L_3, \emptyset\} \).

The value of \( V_1(L_3) \) computed by this equation is 4, corresponding to the observation action \( \{b,c\} \).

At \( t = 0 \), since both events are unobservable, the dynamic programming equation indicates that \( V_0(L_3) = V_1(L_3) \). Therefore the minimum worst case observation cost is \( V_0(L_3) = 4 \).

Table III shows an optimal policy \( g^* = (g_0^*, g_1^*, g_2^*) \) for all information states that are reachable under \( g^* \). Note that, in order to determine which information states were reachable, it was necessary to determine an optimal observation policy for all information states.
VII. LIMITED LOOKAHEAD ALGORITHM

Determining an optimal observation policy using the method described in the previous section can become computationally formidable for large $T$. In this section, we propose a limited lookahead algorithm that approximates an optimal observation policy.

Roughly speaking, in the limited lookahead algorithm a sequence of active acquisition programs are run for a time horizon $T' < T$. Information states at $T'$ are assigned infinite cost only if it is not possible to diagnose $L(G)$ from future observations. This notion of information state diagnosability is formalized in the following definition.

Definition 5. An information state $\pi \in \mathcal{F}_t$ is diagnosable at time $t$ if the cost $V_t(\pi)$ determined by the active acquisition dynamic program is finite. ■

Definition 5 indicates that from a diagnosable information state, the cost-to-go required to diagnose $L(G)$ is finite. The following statement is equivalent to Definition 5:

Theorem 2: Express an information state as $\pi = s_1t_1 + s_2t_2 + \cdots + s_nt_n$, where $\|s_i\| = t$ for $i = 1 \ldots n$. The information state $\pi$ is diagnosable at time $t$ if and only if the language $L_\pi := \tilde{P}(s_1)t_1 + \tilde{P}(s_2)t_2 + \cdots + \tilde{P}(s_n)t_n$ is diagnosable, where $\tilde{P}$ is the projection of $\Sigma$ onto $\Sigma_{uo}$.

Proof. (Sufficiency) Suppose that $V_t(\pi) < \infty$. Then there exists a policy $g = (g_t, g_{t+1}, \ldots, g_{T-1})$ such that the information state $\pi_T$ reached by implementing $g$ along any $t_i$ is certain. The final information state $\pi_T$ consists of those $s_jt_j \in \pi$ that are consistent with the observations made along $t_i$ under policy $g$.

To diagnose $L_\pi$, implement the policy $g' = (0, 0, \ldots, 0, g_t, g_{t+1}, \ldots, g_{T-1})$. Since the first $t$ events along any string in $L_\pi$ are unobservable, along any string $\tilde{P}(s_j)t_j$, the final information state $\pi'_T$ consists of those $\tilde{P}(s_j)t_j$ that are consistent with the observations made along $t_i$. Since the policy $g'$ is identical to $g$ for times greater than $t$, $\tilde{P}(s_j)t_j \in \pi'_T$ if $s_jt_j \in \pi$. Since $\tilde{P}(s_j)t_j$ and $s_jt_j$ contain the same failure events, $\pi'_T$ is certain if $\pi_T$ is certain. Therefore, the policy $g'$ diagnoses $L_\pi$.

(Necessity) We prove necessity by proving the contrapositive statement. Suppose that $V_t(\pi) = \infty$. Then for any $g = (g_t, g_{t+1}, \ldots, g_{T-1})$, there exists a $t_i$ such that the information state $\pi_T$ reached by implementing $g$ along $t_i$ is uncertain.

Select any policy $g' = (g_0, g_1, \ldots, g_{t-1}, g_t, \ldots, g_{T-1})$ and consider the final information state reached by implementing $g'$ along $\tilde{P}(s_j)t_j$. Again, since the first $t$ events along any string in $L_\pi$ are unobservable, the final information state $\pi'_T$ consists of those $\tilde{P}(s_j)t_j$ that are consistent with the observations made along $t_i$. Since $\tilde{P}(s_j)t_j$ and $s_jt_j$ contain the same failures, $\pi'_T$ is uncertain if $\pi_T$ is uncertain. Since for any $g$ we can choose a $t_i$ such that $\pi_T$ is uncertain, for any $g'$ we can choose a $\tilde{P}(s_j)t_j$ such that $\pi'_T$ is uncertain. Therefore $L_\pi$ is not diagnosable. ■

To start the limited lookahead algorithm, we choose a horizon $T' < T$ and consider the $\sigma$-field $\mathcal{F}_{T'}$. For each information state $\pi \in \mathcal{F}_{T'}$, a cost is assigned as follows:

$$V_{T'}(\pi) = \begin{cases} 0 & \text{if } \pi \text{ is diagnosable at } T' \\ \infty & \text{otherwise} \end{cases}$$  (19)

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$g_0$</th>
<th>$g_1$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{aa^b}$</td>
<td>---</td>
<td>---</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\sigma_{ac}$</td>
<td>---</td>
<td>---</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\sigma_{ab} + \sigma_{ab^b}$</td>
<td>---</td>
<td>---</td>
<td>{b}</td>
</tr>
<tr>
<td>$L_3$</td>
<td>$\emptyset$</td>
<td>{b,c}</td>
<td>---</td>
</tr>
</tbody>
</table>

TABLE III
AN OPTIMAL OBSERVATION POLICY FOR DIAGNOSING THE AUTOMATON IN FIGURE 2. ONLY REACHABLE INFORMATION STATES ARE SHOWN.
Fig. 3. An automaton used to illustrate the limited lookahead method.

The cost $V_{T^t}(\pi)$ is assigned to each element in $\mathcal{F}_{T^t}$ by constructing the language $L_\pi$ described in Theorem 2, and then using the result of Theorem 1 to determine whether or not $L_\pi$ is diagnosable.

The dynamic programming equation solved is identical to that in the previous section:

$$V_{t-1}(\pi) = \min_{g(\pi) \in 2^{2^{t-\nu}}} \{c_{g(\pi)} + \max_{\sigma \in \Sigma} V_t(\delta_{g(\pi),t}(\pi, \sigma))\} \text{ for } \pi \in \mathcal{F}_{t-1}, \; t = 1, 2, \ldots, T^t - 1 \quad (20)$$

Once the dynamic program is solved, $V_0(L_{T^t})$ and an observation action $g_0^*(L_{T^t})$ for $t = 0$ are determined. The observer then implements $g_0^*(L_{T^t})$ and calculates the information state at $t = 1$ based on $g_0^*(L_{T^t})$ and its observation.

For $0 < t < t - T'$, the observer generates a sub-$\sigma$-field $G_{T^t+t} \subseteq \mathcal{F}_{T^t+t}$ by considering only those elements in $\mathcal{F}_{T^t+t}$ that are reachable from $\pi_t$, the information state at $t$ resulting from the implementation of policy $g_0^*, g_1^*, \ldots, g_{t-1}^*$ along the system trajectory up to time $t-1$. This sub-$\sigma$-field $G_{T^t+t}$ is defined as:

$$G_{T^t+t} = \{A \in \mathcal{F}_{T^t+t} : A \cap \pi_t = A\} \quad (21)$$

Costs are assigned to each element of $G_{T^t+t}$ as:

$$V_{T^t+t}(\pi) = \begin{cases} 0 & \text{if } \pi \text{ is diagnosable at } T^t + t \\ \infty & \text{otherwise} \end{cases} \quad (22)$$

and then the dynamic program in Equation (20) is used to calculated an observation action for $\pi_t$. The observer then implements that action, calculates a new information state $\pi_{t+1}$, and iterates the algorithm to find an observation action for that information state.

The algorithm finishes when $t = T - T'$ and the observer looks ahead to the final time horizon of the system. The observer implements the policy specified by the solution of the dynamic program (12)-(14) where the horizon is $T - T'$ and the initial information state is $\pi_{T-T'}$.

As an example, consider the automaton in Figure 3, and suppose $\Sigma_{eo} = \{a, b, c, d, e\}$, $\nu_a < \nu_b < \nu_c < \nu_d < \nu_e$ and $T^t = 2$.

At $t = 0$, the observer considers the $\sigma$-field $\mathcal{F}_2 = \sigma(\sigma_{fa}, \sigma_{eb}, \sigma_{fa} + \sigma_{eb})$. Since every element of $\mathcal{F}_2$ is diagnosable at $t = 2$, solving Equation (20) results in the observation action $\emptyset$ at $t = 0$.

At $t = 1$, the information state generated by the observation action at $t = 0$ is necessarily $\pi_1 = L_2$. Consider $G_3 = \{A \in \mathcal{F}_3 : A \cap \pi_1 = A\} = \mathcal{F}_3$. Every element of $G_3$ is diagnosable at $t = 3$, and, as a result of Equation (20), the observation action is $\emptyset$.

At $t = 2$, the information state is $\pi_2 = L_2$, and the observer considers $G_4 = \{A \in \mathcal{F}_4 : A \cap \pi_2 = A\} = \mathcal{F}_4$. The information states $\sigma_{fa} + \sigma_{eb} + \sigma_{eoade}$ are not
diagnosable at \( t = 4 \) and thus have infinite cost. Using Equation (20), we find that an optimal observation action at \( t = 2 \) is to observe \( \{a\} \).

Thus there are two possible information states at \( t = 3 \): if \( a \) is observed when \( t = 2 \), \( \pi_3 = \sigma_f cadd + \sigma_{uo} aade \); otherwise, \( \pi_3 = \sigma_f abee + \sigma_{uo} bbed \).

In the case where \( a \) is observed, the observer generates the \( \sigma \)-field \( G_{5,a} \) using Equation (21):

\[
G_{5,a} = \{\emptyset, \sigma_f cadd, \sigma_{uo} aade, \sigma_f cadd + \sigma_{uo} aade\}
\]

and assigns a cost to each element of \( G_{5,a} \) according to Equation (22); since no further observations can be made after \( t = 5 \), an information state in \( G_{5,a} \) is diagnosable only if it is certain.

By Equation (20), we find that the observation actions are to observe no events when \( t = 3 \) and then to observe \( \{d\} \) when \( t = 4 \). A similar calculation for the case where \( o \) is observed at \( t = 3 \) finds that the same sequence of actions is used there as well.

At each stage of the limited lookahead algorithm, we optimize the worst case \( T_0 \)-step observation cost. The result of this policy is a “procrastinating” diagnoser that makes just enough observations within the lookahead window to ensure that there is some policy that will allow the failure to be diagnosed after the window has passed.

Had the observer used the algorithm of Section 5, it would have determined that the worst-case observation cost is \( 2\nu_o \), which is less than \( \nu_o + \nu_d \).

VIII. ACTIVE ACQUISITION OF INFORMATION FOR STOCHASTIC AUTOMATA

The active acquisition of information problem can be solved for stochastic automata in an analogous manner. The model is identical to that of Section 2, except that the partial transition function \( \delta \) is extended to a state transition probability function \( p \).

Consider a stochastic automation \( G_s \), formally defined as:

\[
G_s = (\Sigma, X, p, x_0),
\]

where

- \( \Sigma \) is a finite set of events
- \( X \) is a finite state space
- \( p : X \times \Sigma \times X \rightarrow [0, 1] \) defines the state transition probability function
- \( x_0 \in X \) is the initial state

As in the logical case, the event set is partitioned into the sets \( \Sigma_{uo}, \Sigma_{fo}, \) and \( \Sigma_{co} \), and again we assume the automaton satisfies (A1) and (A2). The state transition probability function \( p(x_1, e, x_2) \), defined for all events and pairs of states, denotes the probability that, in state \( x_1 \), the event \( e \) will occur and cause a transition to state \( x_2 \). For ease of notation, we also assume that \( p(x_1, e, x_2) > 0 \) for at most one \( x_2 \in X \), and thus define the transition function \( \delta \) as \( \delta(x_1, e) = x_2 \) if \( p(x_1, e, x_2) > 0 \).

The probability that an event \( e \) follows a trace \( s \) is therefore given by:

\[
P(e \mid s) = p(\delta(x_o, s), e)
\]

In the stochastic case, an information state at time \( t \) is a conditional probability mass function (PMF) on \( L_T \), given a sequence of observations \( y^t \) and a sequence of decisions \( u^{t-1} \). The initial information state is the \textit{a priori} PMF of \( L_T \):

\[
P(e_1 e_2 \ldots e_T \mid y^t = \emptyset, u^t = \emptyset) = \prod_{i=1}^{T} p(\delta(x_o, e_1 \ldots e_{i-1}, e_i))
\]

To proceed with the specification of the active acquisition of information problem for stochastic automata we need to make the following modification to Definition 2:
Definition 6. An information state \( \pi \) is called certain if, for all failure types \( F_i \), either every string that has a positive probability in \( \pi \) contains an event in \( \Sigma_f \), or no string that has a positive probability in \( \pi \) contains any event in \( \Sigma_f \).

With the above definition, Definitions 3 and 4 follow with the obvious modifications.

The problem under consideration is to find a policy that preserves diagnosability of the system while minimizing the expected observation cost (as opposed to the worst-case cost in the logical case). For any observation policy \( g \), define its cost function as the expected total observation cost:

\[
J(g) = E[\sum_{t=1}^{T} c_t^g(s)]
\]  

(27)

where \( c_t^g(s) \) denotes the cost of implementing policy \( g \) at time \( t \) along trajectory \( s \).

The active acquisition of information problem for stochastic automata can thus be expressed as:

\textbf{Problem R.} Find a policy \( g^* \in H \) such that

\[
J(g^*) = \inf(J(g) | g \in H)
\]  

(28)

As in the logical case, we call the system diagnosable if there exists a policy that ensures that the final information state reached by implementing that policy is always certain (cf. Definition 6). We solve Problem R by defining an equivalent final condition to Problem Q for all \( \pi \) in \( \mathcal{F}_T \):

\[
V_T(\pi) = \begin{cases} 
\infty & \text{if } \pi \text{ is certain} \\
0 & \text{otherwise} 
\end{cases}
\]  

(29)

The information state transition function updates the probability of each string in \( L_T \) based on the observation action at time \( t \) and the state transition probabilities of the automaton. The probability of each string is updated according to the following equation:

\[
P_t(s_1|s_2, \pi, y_{t+1}, u_t) = \begin{cases} 
\frac{P(e|s_1)P_{t}(s_1|\pi)P(s_2|s_1\epsilon)}{\sum_{s' \in L_T} P(e'|s')P_{t}(s'|\pi)P(s_2|s'|\epsilon)} & \text{if } y_{t+1} = e \\
\frac{P(e'|s')P_{t}(s'|\pi)P(s_2|s'|\epsilon)}{\sum_{s' \in L_T} \sum_{e' \in \Sigma_{ut,unobs}} P(e'|s')P_{t}(s'|\pi)P(s_2|s'|\epsilon)} & \text{if } e \in \Sigma_{ut,unobs} \text{ and } y_{t+1} = \epsilon_o \\
0 & \text{otherwise}
\end{cases}
\]  

(30)

If the event \( e \) is observed at time \( t \), the probability of all traces in \( L_T \) that do not contain \( e \) at time \( t \) must be zero; the probabilities of the remaining traces are computed by normalization. If \( \epsilon_o \) is observed, the probability of all traces where an event observable under our observation action at time \( t \) occurs is zero, and the probabilities of the remaining traces are again computed by normalization.

There is a strong relationship between the stochastic and logical information state transition functions. As events are observed, traces that are not consistent with the observations are eliminated from the logical information state; in the stochastic case, the probability of these traces is set to zero. Thus, at time \( t \), the conditional PMF is always supported on some element of the \( \sigma \)-field \( \mathcal{F}_t \).

Furthermore, given the observations up to \( t \) and the observation actions up to \( t - 1 \), the conditional PMF on the traces of \( L_T \) is uniquely defined and is independent of the observation policy \( g \). (This result is, as expected, consistent with that of Lemma 6.5.10 of [24].)

Problem R can be solved using a dynamic program consisting of Equations (29), (30) and:

\[
V_{t-1}(\pi) = \min_{g(\pi) \in 2^{\Sigma_{co}}} \left\{ c_{g(\pi)} + \sum_{\sigma \in \Omega} V_t(\hat{\delta}_{g(\pi),t}(\pi, \sigma))P(\sigma | \pi(t-1)) \right\} \text{ for } \pi \in \mathcal{F}_{t-1}, t=0,1,\ldots,T
\]  

(31)
IX. DISCUSSION

This paper provides a framework for formulating and solving the active acquisition problem in discrete-event systems. By identifying an appropriate filtration of information $\sigma$-fields, the problem of finding an observation policy that minimizes a worst-case observation cost or an average observation cost while preserving diagnosability can be systematically solved using a dynamic program for both logical and stochastic models. A limited lookahead algorithm that allows for computational savings has also been presented.

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